SET 7
ESERCIZIO

#Calcolare il numero dei grafi su 5 vertici e scegliere per ogni grafo (G1, G2, G3, ...) una matrice di adiacenza (A1, A2, A3, ...) nella classe di quelle che danno grafi isomorfi.

Dobbiamo studiare la rappresentazione del gruppo S5 sulle permutazioni delle posizioni triangolari superiori di una matrice 5x5

0 1 2 3 4
1 0 5 6 7
2 5 0 8 9
3 6 8 0 10
4 7 9 10 0

La permutazione (1,2) in S5 corrisponde a (2,5)(3,6)(7,4) mentre (1,2,3,4,5) corrisponde a (1,5,8,10,4)(2,6,9,3,7). Creiamo poi il gruppo “g” come gruppo generato dalle due permutazioni. Dopodichè calcoliamo la cardinalità dell’insieme delle orbite delle combinazioni dei numeri [1..10] sul gruppo S5.

```
gap> LoadPackage(“Grape”);
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Loading  GRAPE 4.8 (GRaph Algorithms using PErmutation groups)
by Leonard H. Soicher (http://www.maths.qmul.ac.uk/~lsoicher/).
Homepage: http://www.maths.qmul.ac.uk/~lsoicher/grape/
────────────────────────────────────────────────────────────────────────────

true
gap> p:=(2,5)(3,6)(4,7);
(2,5)(3,6)(4,7)
gap> q:=(1,4,10,8,5)(2,7,3,9,6);
(1,4,10,8,5)(2,7,3,9,6)
gap> g:=Group(p,q);
Group([ (2,5)(3,6)(4,7), (1,4,10,8,5)(2,7,3,9,6) ])
gap> Size(Orbits(g,Combinations([1,2,3,4,5,6,7,8,9,10]),OnSets));
34
```

Il numero dei grafi non isomorfi su 5 vertici è 34.

Dobbiamo ora scegliere per ogni grafo una matrice di adiacenza nella classe di quelle che danno grafi isomorfi.

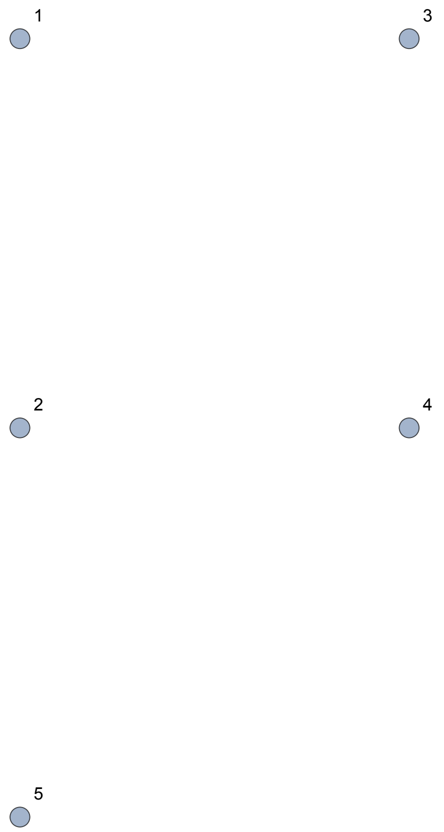
```
gap> Orbit(g,[],OnSets);
[ [  ] ]
gap> Size(Orbit(g,[],OnSets));
1
```

```
In[1]:= Binomial[10, 0]
Out[1]= 1

In[1197]:= L1 = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
Out[1197]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[1204]:= A1 = {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
Out[1204]= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[4]:= MatrixForm[A1]
```

In[1205]:= $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

```
Out[1205]= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1207]:= G1 = AdjacencyGraph[A1, VertexLabels → "Name"]
```

Out[1207]=




```
gap> Orbit(g,[1],OnSets);
[ [ 1 ], [ 4 ], [ 7 ], [ 10 ], [ 3 ], [ 8 ], [ 6 ], [ 9 ], [ 5 ], [ 2 ] ]
gap> Size(Orbit(g,[1],OnSets));
10
```

```
In[43]:= Binomial[10, 1]
Out[43]= 10
```

In[1208]:= **L2 = {1, 0, 0, 0, 0, 0, 0, 0, 0, 0}**

Out[1208]= {1, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[1209]:= **A2 = {{0, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}**

Out[1209]= {{0, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1210]:= **MatrixForm[A2]**

Out[1210]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1218]:= **G2 = AdjacencyGraph[A2, VertexLabels → "Name"]**

Out[1218]=


```
gap> Orbit(g,[1,2],OnSets);
[ [ 1, 2 ], [ 1, 5 ], [ 4, 7 ], [ 1, 4 ], [ 3, 10 ], [ 1, 7 ], [ 4, 10 ], [ 6, 10 ], [ 8, 9 ], [ 3, 4 ], [ 7, 10 ],
  [ 8, 10 ], [ 2, 8 ], [ 5, 6 ], [ 6, 7 ], [ 9, 10 ], [ 3, 8 ], [ 5, 8 ], [ 5, 7 ], [ 2, 3 ], [ 6, 8 ], [ 5, 9 ],
  [ 2, 4 ], [ 1, 3 ], [ 7, 9 ], [ 2, 5 ], [ 2, 9 ], [ 1, 6 ], [ 4, 9 ], [ 3, 6 ] ]
gap> Size(Orbit(g,[1,2],OnSets));
30
gap> Orbit(g,[1,8],OnSets);
[ [ 1, 8 ], [ 4, 5 ], [ 2, 7 ], [ 1, 10 ], [ 3, 7 ], [ 4, 8 ], [ 4, 6 ], [ 3, 9 ], [ 7, 8 ], [ 5, 10 ], [ 2, 10 ],
  [ 6, 9 ], [ 3, 5 ], [ 2, 6 ], [ 1, 9 ] ]
gap> Size(Orbit(g,[1,8],OnSets));
15
```

In[48]:= **Binomial[10, 2]**

Out[48]= 45

In[49]:= **Binomial[10, 2] == (30 + 15)**

Out[49]= True

In[1212]:= **L3 = {1, 1, 0, 0, 0, 0, 0, 0, 0, 0}**

Out[1212]= {1, 1, 0, 0, 0, 0, 0, 0, 0, 0}

In[1219]:= **A3 = {{0, 1, 1, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}**

Out[1219]= {{0, 1, 1, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

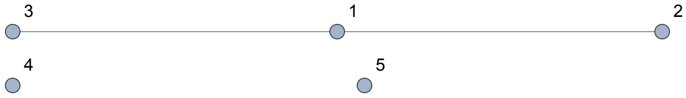
In[1224]:= **MatrixForm[%]**

Out[1224]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1225]:= `G3 = AdjacencyGraph[A3, VertexLabels → "Name"]`

Out[1225]=



In[1226]:= `L4 = {1, 0, 0, 0, 0, 0, 0, 1, 0, 0}`

Out[1226]= `{1, 0, 0, 0, 0, 0, 0, 1, 0, 0}`

In[1227]:= `A4 = {{0, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 0, 0}}`

Out[1227]= `{{0, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 0, 0}}`

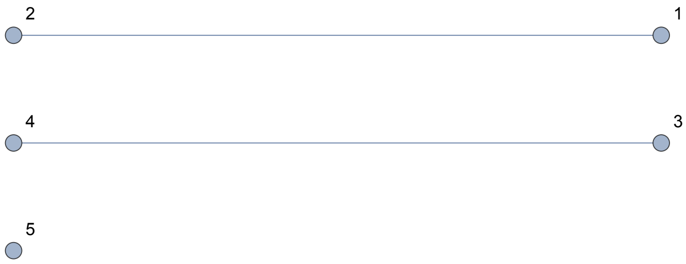
In[1228]:= `MatrixForm[A4]`

Out[1228]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1229]:= `G4 = AdjacencyGraph[A4, VertexLabels → "Name"]`

Out[1229]=




```
gap> Orbit(g,[1,2,3],OnSets);
[ [ 1, 2, 3 ], [ 1, 5, 6 ], [ 4, 7, 9 ], [ 1, 2, 4 ], [ 3, 6, 10 ], [ 1, 5, 7 ], [ 4, 7, 10 ], [ 2, 8, 9 ],
  [ 1, 3, 4 ], [ 3, 8, 10 ], [ 5, 8, 9 ], [ 5, 6, 7 ], [ 1, 6, 7 ], [ 4, 9, 10 ], [ 6, 8, 10 ], [ 2, 3, 4 ],
  [ 7, 9, 10 ], [ 2, 5, 8 ], [ 3, 6, 8 ], [ 2, 5, 9 ] ]
gap> Size(Orbit(g,[1,2,3],OnSets));
20
gap> Orbit(g,[1,2,5],OnSets);
[ [ 1, 2, 5 ], [ 1, 4, 7 ], [ 3, 4, 10 ], [ 6, 7, 10 ], [ 8, 9, 10 ], [ 2, 3, 8 ], [ 5, 6, 8 ], [ 5, 7, 9 ],
  [ 2, 4, 9 ], [ 1, 3, 6 ] ]
gap> Size(Orbit(g,[1,2,5],OnSets));
10
gap> Orbit(g,[1,2,6],OnSets);
[ [ 1, 2, 6 ], [ 1, 3, 5 ], [ 2, 4, 7 ], [ 1, 4, 9 ], [ 4, 5, 7 ], [ 3, 7, 10 ], [ 1, 7, 9 ], [ 4, 6, 10 ],
  [ 1, 3, 10 ], [ 3, 8, 9 ], [ 3, 4, 6 ], [ 2, 8, 10 ], [ 1, 6, 10 ], [ 4, 8, 9 ], [ 6, 8, 9 ], [ 5, 6, 9 ],
  [ 3, 6, 7 ], [ 2, 9, 10 ], [ 5, 8, 10 ], [ 5, 7, 8 ], [ 2, 4, 8 ], [ 7, 8, 9 ], [ 5, 6, 10 ], [ 2, 5, 6 ],
  [ 2, 3, 9 ], [ 5, 9, 10 ], [ 6, 7, 8 ], [ 1, 5, 8 ], [ 5, 7, 10 ], [ 3, 5, 6 ], [ 2, 3, 10 ], [ 1, 2, 8 ],
  [ 2, 3, 5 ], [ 1, 2, 7 ], [ 6, 7, 9 ], [ 1, 6, 8 ], [ 3, 4, 8 ], [ 1, 4, 5 ], [ 2, 4, 10 ], [ 1, 3, 8 ],
  [ 2, 3, 6 ], [ 1, 2, 9 ], [ 3, 4, 7 ], [ 3, 4, 9 ], [ 2, 4, 5 ], [ 1, 4, 10 ], [ 7, 8, 10 ], [ 4, 5, 9 ],
  [ 2, 7, 9 ], [ 1, 5, 9 ], [ 4, 6, 7 ], [ 3, 9, 10 ], [ 6, 9, 10 ], [ 2, 5, 7 ], [ 1, 7, 10 ], [ 4, 8, 10 ],
  [ 3, 5, 8 ], [ 1, 4, 6 ], [ 2, 6, 8 ], [ 1, 3, 7 ] ]
gap> Size(Orbit(g,[1,2,6],OnSets));
60
gap> Orbit(g,[1,2,10],OnSets);
[ [ 1, 2, 10 ], [ 1, 5, 10 ], [ 4, 7, 8 ], [ 1, 4, 8 ], [ 3, 5, 10 ], [ 1, 7, 8 ], [ 4, 5, 10 ], [ 2, 6, 10 ],
```

```
 [ 1, 8, 9 ], [ 3, 4, 5 ], [ 2, 7, 10 ], [ 1, 8, 10 ], [ 2, 7, 8 ], [ 4, 5, 6 ], [ 2, 6, 7 ], [ 1, 9, 10 ],
 [ 3, 7, 8 ], [ 4, 5, 8 ], [ 3, 5, 7 ], [ 2, 3, 7 ], [ 4, 6, 8 ], [ 3, 5, 9 ], [ 2, 4, 6 ], [ 1, 3, 9 ],
 [ 3, 7, 9 ], [ 2, 5, 10 ], [ 2, 6, 9 ], [ 1, 6, 9 ], [ 4, 6, 9 ], [ 3, 6, 9 ] ]
gap> Size(Orbit(g,[1,2,10],OnSets));
30
```

In[58]:= **Binomial[10, 3]**

Out[58]= 120

In[72]:= **Binomial[10, 3] == (20 + 10 + 60 + 30)**

Out[72]= True

In[1230]:= **L5 = {1, 1, 1, 0, 0, 0, 0, 0, 0, 0}**

Out[1230]= {1, 1, 1, 0, 0, 0, 0, 0, 0, 0}

In[1241]:= **A5 = {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}**

Out[1241]= {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

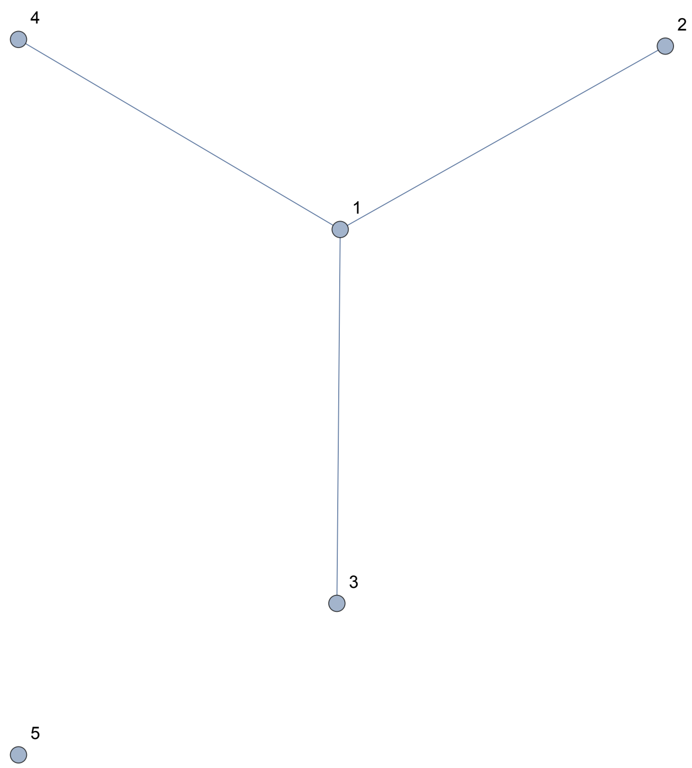
In[1234]:= **MatrixForm[%]**

Out[1234]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1243]:= **G5 = AdjacencyGraph[A5, VertexLabels → "Name"]**

Out[1243]=



In[1244]:= **L6 = {1, 1, 0, 0, 1, 0, 0, 0, 0, 0}**

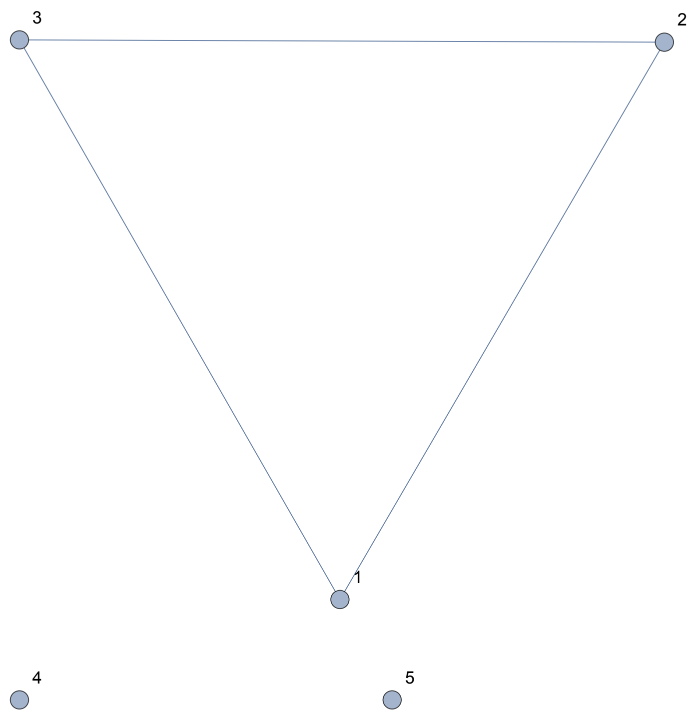
Out[1244]= {1, 1, 0, 0, 1, 0, 0, 0, 0, 0}

In[1245]:=
```
A6 = {{0, 1, 1, 0, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```
Out[1245]=
```
{{0, 1, 1, 0, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

In[1246]:=
```
MatrixForm[A6]
```
Out[1246]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1247]:=
```
G6 = AdjacencyGraph[A6, VertexLabels → "Name"]
```
Out[1247]=



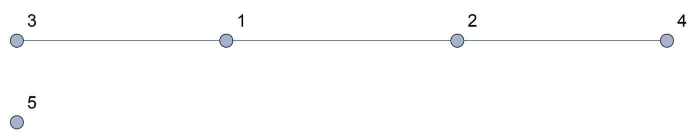
In[1248]:=
```
L7 = {1, 1, 0, 0, 0, 1, 0, 0, 0, 0}
```
Out[1248]=
```
{1, 1, 0, 0, 0, 1, 0, 0, 0, 0}
```

In[1249]:=
```
A7 = {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 0}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}}
```
Out[1249]=
```
{{0, 1, 1, 0, 0}, {1, 0, 0, 1, 0}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

In[1250]:=
```
MatrixForm[A7]
```
Out[1250]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1251]:=
```
G7 = AdjacencyGraph[A7, VertexLabels → "Name"]
```
Out[1251]=



In[1252]:= **L8 = {1, 1, 0, 0, 0, 0, 0, 0, 0, 1}**

Out[1252]= {1, 1, 0, 0, 0, 0, 0, 0, 0, 1}

In[1253]:= **A8 = {{0, 1, 1, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 1, 0}}**

Out[1253]= {{0, 1, 1, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 1, 0}}

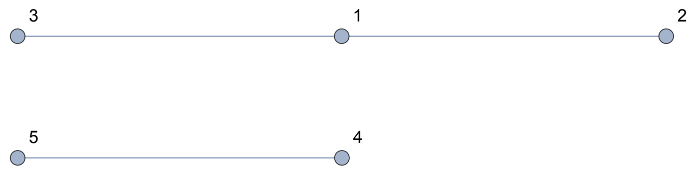
In[1254]:= **MatrixForm[A8]**

Out[1254]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[1255]:= **G8 = AdjacencyGraph[A8, VertexLabels → "Name"]**

Out[1255]=




```
gap> Orbit(g,[1,2,3,4],OnSets);
[ [ 1, 2, 3, 4 ], [ 1, 5, 6, 7 ], [ 4, 7, 9, 10 ], [ 3, 6, 8, 10 ], [ 2, 5, 8, 9 ] ]
gap> Size(Orbit(g,[1,2,3,4],OnSets));
5
gap> Orbit(g,[1,2,3,5],OnSets);
[ [ 1, 2, 3, 5 ], [ 1, 2, 5, 6 ], [ 1, 4, 7, 9 ], [ 1, 2, 4, 7 ], [ 3, 4, 6, 10 ], [ 1, 4, 5, 7 ], [ 3, 4, 7, 10 ],
  [ 3, 6, 7, 10 ], [ 2, 8, 9, 10 ], [ 1, 3, 4, 10 ], [ 4, 6, 7, 10 ], [ 3, 8, 9, 10 ], [ 2, 3, 8, 9 ],
  [ 5, 8, 9, 10 ], [ 5, 6, 7, 8 ], [ 1, 6, 7, 10 ], [ 4, 8, 9, 10 ], [ 2, 3, 8, 10 ], [ 6, 8, 9, 10 ],
  [ 5, 6, 8, 9 ], [ 5, 6, 7, 9 ], [ 1, 5, 6, 8 ], [ 2, 3, 4, 8 ], [ 7, 8, 9, 10 ], [ 5, 6, 8, 10 ], [ 5, 7, 8, 9 ],
  [ 2, 5, 6, 8 ], [ 2, 3, 4, 9 ], [ 1, 2, 3, 6 ], [ 1, 2, 3, 8 ], [ 1, 2, 4, 5 ], [ 5, 7, 9, 10 ], [ 3, 5, 6, 8 ],
  [ 1, 2, 5, 8 ], [ 2, 4, 8, 9 ], [ 1, 3, 5, 6 ], [ 2, 3, 5, 8 ], [ 1, 2, 5, 7 ], [ 6, 7, 9, 10 ], [ 2, 4, 7, 9 ],
  [ 4, 5, 7, 9 ], [ 1, 4, 7, 10 ], [ 2, 4, 9, 10 ], [ 1, 3, 6, 8 ], [ 2, 3, 6, 8 ], [ 1, 2, 5, 9 ], [ 5, 6, 7, 10 ],
  [ 1, 2, 4, 9 ], [ 1, 5, 7, 9 ], [ 1, 3, 4, 7 ], [ 3, 4, 9, 10 ], [ 1, 3, 6, 10 ], [ 3, 4, 8, 10 ],
  [ 6, 7, 8, 10 ], [ 2, 4, 5, 9 ], [ 2, 5, 7, 9 ], [ 1, 4, 6, 7 ], [ 2, 3, 4, 10 ], [ 1, 3, 4, 6 ], [ 1, 3, 6, 7 ] ]
gap> Size(Orbit(g,[1,2,3,5],OnSets));
60
gap> Orbit(g,[1,2,3,7],OnSets);
[ [ 1, 2, 3, 7 ], [ 1, 4, 5, 6 ], [ 3, 4, 7, 9 ], [ 1, 2, 4, 10 ], [ 4, 6, 7, 9 ], [ 3, 6, 9, 10 ], [ 1, 5, 7, 10 ],
  [ 4, 7, 8, 10 ], [ 2, 3, 6, 10 ], [ 2, 6, 8, 9 ], [ 1, 3, 4, 8 ], [ 3, 5, 8, 10 ], [ 3, 5, 6, 10 ],
  [ 2, 7, 8, 9 ], [ 3, 5, 8, 9 ], [ 2, 5, 6, 7 ], [ 1, 6, 7, 8 ], [ 4, 5, 9, 10 ], [ 2, 6, 8, 10 ], [ 1, 5, 8, 9 ],
  [ 1, 2, 8, 9 ], [ 4, 5, 8, 9 ], [ 3, 5, 6, 7 ], [ 1, 5, 6, 9 ], [ 2, 3, 4, 5 ], [ 2, 7, 9, 10 ], [ 1, 6, 8, 10 ],
  [ 2, 5, 7, 8 ], [ 4, 5, 6, 7 ], [ 1, 5, 6, 10 ], [ 2, 3, 4, 6 ], [ 1, 2, 3, 9 ], [ 1, 2, 4, 6 ], [ 1, 7, 9, 10 ],
  [ 3, 6, 7, 8 ], [ 1, 3, 8, 10 ], [ 2, 4, 5, 8 ], [ 1, 3, 5, 7 ], [ 2, 3, 4, 7 ], [ 1, 2, 3, 10 ], [ 1, 2, 4, 8 ],
  [ 2, 4, 7, 10 ], [ 1, 4, 9, 10 ], [ 3, 4, 6, 8 ], [ 2, 3, 5, 9 ], [ 1, 3, 4, 9 ], [ 3, 7, 9, 10 ],
  [ 4, 7, 8, 9 ], [ 1, 5, 7, 8 ], [ 4, 5, 7, 10 ], [ 3, 7, 8, 10 ], [ 4, 6, 8, 10 ], [ 2, 5, 9, 10 ],
  [ 2, 5, 6, 9 ], [ 1, 6, 7, 9 ], [ 4, 6, 9, 10 ], [ 3, 6, 8, 9 ], [ 1, 3, 4, 5 ], [ 2, 5, 8, 10 ], [ 1, 2, 6, 7 ] ]
gap> Size(Orbit(g,[1,2,3,7],OnSets));
60
gap> Orbit(g,[1,2,5,10],OnSets);
[ [ 1, 2, 5, 10 ], [ 1, 4, 7, 8 ], [ 3, 4, 5, 10 ], [ 2, 6, 7, 10 ], [ 1, 8, 9, 10 ], [ 2, 3, 7, 8 ],
  [ 4, 5, 6, 8 ], [ 3, 5, 7, 9 ], [ 2, 4, 6, 9 ], [ 1, 3, 6, 9 ] ]
```

```
gap> Size(Orbit(g,[1,2,5,10],OnSets));
10
gap> Orbit(g,[1,2,6,7],OnSets);
[ [ 1, 2, 6, 7 ], [ 1, 3, 4, 5 ], [ 2, 3, 4, 7 ], [ 1, 4, 9, 10 ], [ 4, 5, 6, 7 ], [ 3, 7, 9, 10 ], [ 1, 7, 9, 10 ],
  [ 4, 6, 8, 10 ], [ 1, 2, 3, 10 ], [ 4, 6, 9, 10 ], [ 3, 6, 8, 9 ], [ 3, 4, 6, 8 ], [ 3, 7, 8, 10 ],
  [ 2, 5, 8, 10 ], [ 1, 5, 6, 10 ], [ 4, 7, 8, 9 ], [ 2, 6, 8, 10 ], [ 2, 5, 6, 9 ], [ 3, 6, 7, 8 ],
  [ 2, 5, 9, 10 ], [ 3, 5, 8, 9 ], [ 1, 5, 7, 8 ], [ 1, 2, 4, 8 ], [ 3, 5, 6, 10 ], [ 3, 5, 8, 10 ],
  [ 2, 5, 7, 8 ], [ 2, 3, 5, 9 ], [ 1, 6, 7, 8 ], [ 2, 6, 8, 9 ], [ 1, 5, 6, 9 ], [ 4, 5, 7, 10 ], [ 2, 3, 6, 10 ],
  [ 1, 2, 8, 9 ], [ 1, 5, 8, 9 ], [ 2, 4, 5, 8 ], [ 1, 3, 5, 7 ], [ 1, 6, 7, 9 ], [ 1, 3, 4, 8 ], [ 2, 3, 4, 5 ],
  [ 2, 5, 6, 7 ], [ 1, 2, 3, 9 ], [ 1, 2, 4, 6 ], [ 2, 4, 7, 10 ], [ 1, 3, 8, 10 ], [ 2, 7, 8, 9 ], [ 1, 4, 5, 6 ],
  [ 1, 5, 7, 10 ], [ 1, 3, 4, 9 ], [ 2, 3, 4, 6 ], [ 4, 5, 9, 10 ], [ 1, 2, 3, 7 ], [ 4, 6, 7, 9 ], [ 1, 6, 8, 10 ],
  [ 4, 5, 8, 9 ], [ 3, 5, 6, 7 ], [ 1, 2, 4, 10 ], [ 2, 7, 9, 10 ], [ 3, 4, 7, 9 ], [ 4, 7, 8, 10 ],
  [ 3, 6, 9, 10 ] ]
gap> Size(Orbit(g,[1,2,6,7],OnSets));
60
gap> Orbit(g,[1,2,6,8],OnSets);
[ [ 1, 2, 6, 8 ], [ 1, 3, 5, 8 ], [ 2, 4, 5, 7 ], [ 1, 4, 5, 9 ], [ 1, 3, 7, 10 ], [ 1, 2, 7, 9 ], [ 1, 4, 6, 10 ],
  [ 3, 4, 8, 9 ], [ 3, 4, 6, 7 ], [ 2, 4, 8, 10 ], [ 6, 7, 8, 9 ], [ 5, 6, 9, 10 ], [ 2, 3, 9, 10 ],
  [ 5, 7, 8, 10 ], [ 2, 3, 5, 6 ] ]
gap> Size(Orbit(g,[1,2,6,8],OnSets));
15
```

In[77]:= **Binomial[10, 4]**

Out[77]= 210

In[100]:= **Binomial[10, 4] == (5 + 60 + 60 + 10 + 60 + 15)**

Out[100]= True

In[1256]:= **L9 = {1, 1, 1, 1, 0, 0, 0, 0, 0, 0}**

Out[1256]= {1, 1, 1, 1, 0, 0, 0, 0, 0, 0}

In[1257]:= **A9 = {{0, 1, 1, 1, 1}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}}**

Out[1257]= {{0, 1, 1, 1, 1}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}}

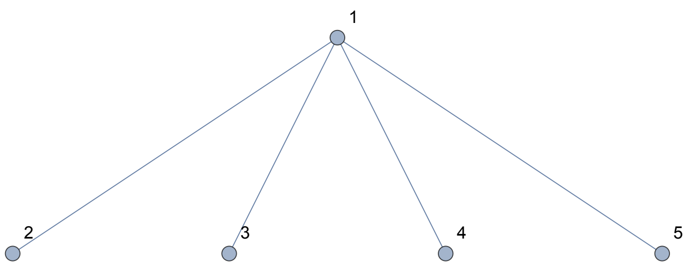
In[1258]:= **MatrixForm[A9]**

Out[1258]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1259]:= **G9 = AdjacencyGraph[A9, VertexLabels → "Name"]**

Out[1259]=



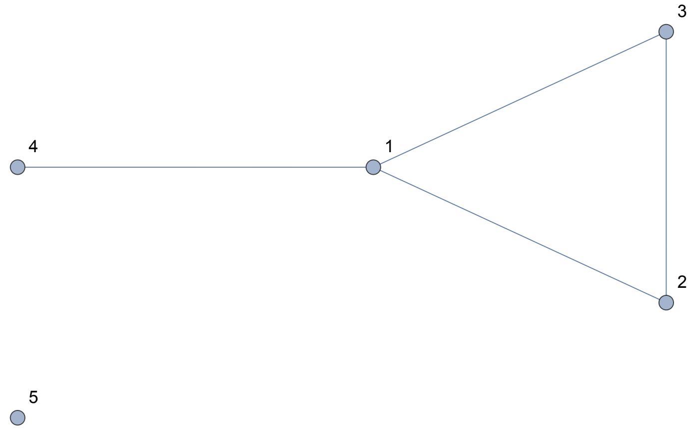
In[1260]:=
```
L10 = {1, 1, 1, 0, 1, 0, 0, 0, 0, 0}
```
Out[1260]=
```
{1, 1, 1, 0, 1, 0, 0, 0, 0, 0}
```

In[1261]:=
```
A10 = {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```
Out[1261]=
```
{{0, 1, 1, 1, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

In[1262]:=
```
MatrixForm[A10]
```
Out[1262]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1263]:=
```
G10 = AdjacencyGraph[A10, VertexLabels → "Name"]
```
Out[1263]=



In[1264]:=
```
L11 = {1, 1, 1, 0, 0, 0, 1, 0, 0, 0}
```
Out[1264]=
```
{1, 1, 1, 0, 0, 0, 1, 0, 0, 0}
```

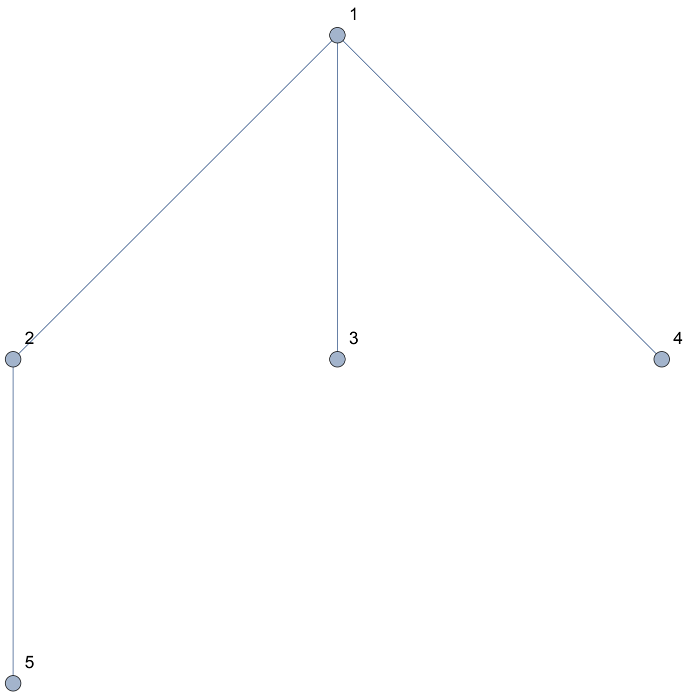
In[1265]:=
```
A11 = {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}}
```
Out[1265]=
```
{{0, 1, 1, 1, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}}
```

In[1267]:=
```
MatrixForm[A11]
```
Out[1267]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1268]:= **G11 = AdjacencyGraph[A11, VertexLabels → "Name"]**

Out[1268]=



In[1269]:= **L12 = {1, 1, 0, 0, 1, 0, 0, 0, 0, 1}**

Out[1269]= {1, 1, 0, 0, 1, 0, 0, 0, 0, 1}

In[1270]:= **A12 = {{0, 1, 1, 0, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 1, 0}}**

Out[1270]= {{0, 1, 1, 0, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 1, 0}}

In[1271]:= **MatrixForm[A12]**

Out[1271]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[1272]:= **G12 = AdjacencyGraph[A12, VertexLabels → "Name"]**

Out[1272]=

In[1273]:= **L13 = {1, 1, 0, 0, 0, 1, 1, 0, 0, 0}**

Out[1273]= {1, 1, 0, 0, 0, 1, 1, 0, 0, 0}

In[1274]:= **A13 = {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 1}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 1, 0, 0, 0}}**

Out[1274]= {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 1}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 1, 0, 0, 0}}

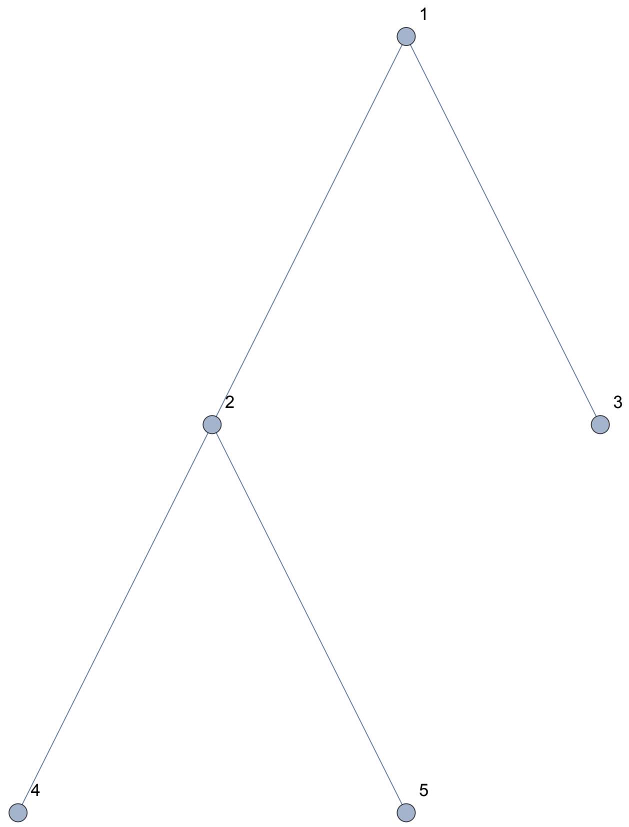
In[1275]:= **MatrixForm[A13]**

Out[1275]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1276]:= **G13 = AdjacencyGraph[A13, VertexLabels → "Name"]**

Out[1276]=



In[1277]:= **L14 = {1, 1, 0, 0, 0, 1, 0, 1, 0, 0}**

Out[1277]= {1, 1, 0, 0, 0, 1, 0, 1, 0, 0}

In[1278]:= **A14 = {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 0}, {1, 0, 0, 1, 0}, {0, 1, 1, 0, 0}, {0, 0, 0, 0, 0}}**

Out[1278]= {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 0}, {1, 0, 0, 1, 0}, {0, 1, 1, 0, 0}, {0, 0, 0, 0, 0}}

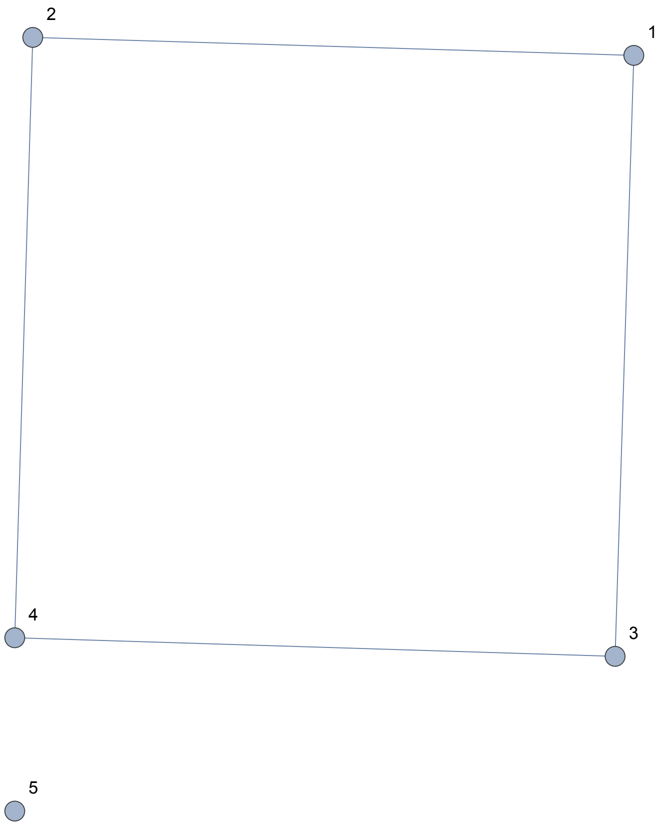
In[1279]:= **MatrixForm[A14]**

Out[1279]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1280]:= **G14 = AdjacencyGraph[A14, VertexLabels → "Name"]**

Out[1280]=




```
gap> Orbit(g,[1,2,3,4,5],OnSets);
[ [ 1, 2, 3, 4, 5 ], [ 1, 2, 5, 6, 7 ], [ 1, 4, 7, 9, 10 ], [ 1, 2, 3, 4, 7 ], [ 3, 4, 6, 8, 10 ],
  [ 1, 4, 5, 6, 7 ], [ 3, 4, 7, 9, 10 ], [ 3, 6, 7, 8, 10 ], [ 2, 5, 8, 9, 10 ], [ 1, 2, 3, 4, 10 ],
  [ 4, 6, 7, 9, 10 ], [ 3, 6, 8, 9, 10 ], [ 2, 3, 5, 8, 9 ], [ 1, 5, 6, 7, 8 ], [ 1, 5, 6, 7, 10 ],
  [ 4, 7, 8, 9, 10 ], [ 2, 3, 6, 8, 10 ], [ 2, 5, 6, 8, 9 ], [ 1, 5, 6, 7, 9 ], [ 1, 2, 3, 4, 8 ],
  [ 3, 5, 6, 8, 10 ], [ 2, 5, 7, 8, 9 ], [ 1, 2, 3, 4, 9 ], [ 1, 2, 3, 4, 6 ], [ 4, 5, 7, 9, 10 ],
  [ 1, 2, 5, 8, 9 ], [ 2, 4, 5, 8, 9 ], [ 1, 3, 5, 6, 7 ], [ 2, 4, 7, 9, 10 ], [ 1, 3, 6, 8, 10 ] ]
gap> Size(Orbit(g,[1,2,3,4,5],OnSets));
30
gap> Orbit(g,[1,2,3,5,6],OnSets);
[ [ 1, 2, 3, 5, 6 ], [ 1, 2, 4, 7, 9 ], [ 1, 4, 5, 7, 9 ], [ 3, 4, 6, 7, 10 ], [ 1, 3, 4, 6, 10 ],
  [ 2, 3, 8, 9, 10 ], [ 1, 3, 6, 7, 10 ], [ 2, 4, 8, 9, 10 ], [ 5, 6, 8, 9, 10 ], [ 5, 6, 7, 8, 9 ],
  [ 2, 3, 4, 8, 9 ], [ 5, 7, 8, 9, 10 ], [ 5, 6, 7, 8, 10 ], [ 1, 2, 5, 6, 8 ], [ 5, 6, 7, 9, 10 ],
  [ 1, 3, 5, 6, 8 ], [ 2, 3, 4, 8, 10 ], [ 1, 2, 3, 5, 8 ], [ 1, 2, 4, 5, 7 ], [ 2, 3, 4, 9, 10 ],
  [ 1, 2, 3, 6, 8 ], [ 1, 2, 4, 5, 9 ], [ 1, 3, 4, 7, 10 ], [ 6, 7, 8, 9, 10 ], [ 2, 4, 5, 7, 9 ],
  [ 1, 2, 5, 7, 9 ], [ 1, 4, 6, 7, 10 ], [ 3, 4, 8, 9, 10 ], [ 2, 3, 5, 6, 8 ], [ 1, 3, 4, 6, 7 ] ]
gap> Size(Orbit(g,[1,2,3,5,6],OnSets));
30
gap> Orbit(g,[1,2,3,5,7],OnSets);
[ [ 1, 2, 3, 5, 7 ], [ 1, 2, 4, 5, 6 ], [ 1, 3, 4, 7, 9 ], [ 1, 2, 4, 7, 10 ], [ 1, 4, 6, 7, 9 ],
  [ 3, 4, 6, 9, 10 ], [ 1, 4, 5, 7, 10 ], [ 3, 4, 7, 8, 10 ], [ 2, 3, 4, 6, 10 ], [ 3, 6, 7, 9, 10 ],
  [ 2, 6, 8, 9, 10 ], [ 1, 3, 4, 8, 10 ], [ 4, 6, 7, 8, 10 ], [ 3, 5, 8, 9, 10 ], [ 3, 5, 6, 7, 10 ],
  [ 2, 7, 8, 9, 10 ], [ 2, 3, 6, 8, 9 ], [ 2, 5, 6, 7, 8 ], [ 1, 6, 7, 8, 10 ], [ 4, 5, 8, 9, 10 ],
  [ 2, 3, 5, 8, 10 ], [ 1, 5, 6, 8, 9 ], [ 1, 2, 3, 8, 9 ], [ 3, 5, 6, 7, 8 ], [ 3, 5, 6, 8, 9 ], [ 2, 5, 6, 7, 9 ],
  [ 2, 3, 4, 5, 8 ], [ 1, 5, 6, 8, 10 ], [ 2, 5, 6, 8, 10 ], [ 1, 5, 7, 8, 9 ], [ 4, 5, 6, 7, 9 ],
  [ 2, 3, 4, 6, 8 ], [ 1, 2, 3, 5, 9 ], [ 1, 2, 5, 6, 9 ], [ 2, 3, 4, 5, 9 ], [ 1, 2, 3, 6, 7 ], [ 1, 5, 7, 9, 10 ],
```

```
  [ 1, 2, 3, 8, 10 ], [ 1, 2, 4, 5, 8 ], [ 1, 2, 5, 7, 8 ], [ 1, 2, 4, 8, 9 ], [ 1, 3, 4, 5, 6 ], [ 2, 3, 4, 7, 9 ],
  [ 1, 2, 3, 6, 10 ], [ 2, 5, 7, 9, 10 ], [ 1, 2, 4, 6, 7 ], [ 1, 6, 7, 9, 10 ], [ 1, 2, 4, 9, 10 ],
  [ 1, 3, 4, 6, 8 ], [ 4, 5, 7, 8, 9 ], [ 1, 3, 4, 5, 7 ], [ 4, 5, 6, 7, 10 ], [ 1, 3, 5, 6, 10 ],
  [ 2, 4, 7, 8, 9 ], [ 2, 4, 5, 9, 10 ], [ 1, 3, 6, 7, 8 ], [ 2, 3, 4, 7, 10 ], [ 1, 3, 4, 9, 10 ],
  [ 3, 7, 8, 9, 10 ], [ 4, 6, 8, 9, 10 ] ]
gap> Size(Orbit(g,[1,2,3,5,7],OnSets));
60
gap> Orbit(g,[1,2,3,5,10],OnSets);
[ [ 1, 2, 3, 5, 10 ], [ 1, 2, 5, 6, 10 ], [ 1, 4, 7, 8, 9 ], [ 1, 2, 4, 7, 8 ], [ 3, 4, 5, 6, 10 ],
  [ 1, 4, 5, 7, 8 ], [ 3, 4, 5, 7, 10 ], [ 2, 3, 6, 7, 10 ], [ 1, 2, 8, 9, 10 ], [ 1, 3, 4, 5, 10 ],
  [ 2, 4, 6, 7, 10 ], [ 1, 3, 8, 9, 10 ], [ 2, 3, 7, 8, 9 ], [ 1, 5, 8, 9, 10 ], [ 4, 5, 6, 7, 8 ],
  [ 1, 2, 6, 7, 10 ], [ 1, 4, 8, 9, 10 ], [ 2, 3, 7, 8, 10 ], [ 1, 6, 8, 9, 10 ], [ 4, 5, 6, 8, 9 ],
  [ 3, 5, 6, 7, 9 ], [ 1, 4, 5, 6, 8 ], [ 2, 3, 4, 7, 8 ], [ 1, 7, 8, 9, 10 ], [ 4, 5, 6, 8, 10 ],
  [ 3, 5, 7, 8, 9 ], [ 2, 4, 5, 6, 8 ], [ 2, 3, 4, 6, 9 ], [ 1, 2, 3, 6, 9 ], [ 1, 2, 3, 7, 8 ], [ 1, 2, 4, 5, 10 ],
  [ 3, 5, 7, 9, 10 ], [ 3, 4, 5, 6, 8 ], [ 1, 2, 5, 8, 10 ], [ 2, 4, 6, 8, 9 ], [ 1, 3, 5, 6, 9 ],
  [ 2, 3, 5, 7, 8 ], [ 1, 2, 5, 7, 10 ], [ 2, 6, 7, 9, 10 ], [ 2, 4, 6, 7, 9 ], [ 3, 4, 5, 7, 9 ],
  [ 1, 4, 7, 8, 10 ], [ 2, 4, 6, 9, 10 ], [ 1, 3, 6, 8, 9 ], [ 2, 3, 6, 7, 8 ], [ 1, 2, 5, 9, 10 ],
  [ 2, 5, 6, 7, 10 ], [ 1, 2, 4, 6, 9 ], [ 1, 3, 5, 7, 9 ], [ 1, 3, 4, 7, 8 ], [ 3, 4, 5, 9, 10 ],
  [ 1, 3, 6, 9, 10 ], [ 3, 4, 5, 8, 10 ], [ 2, 6, 7, 8, 10 ], [ 2, 4, 5, 6, 9 ], [ 2, 3, 5, 7, 9 ],
  [ 1, 4, 6, 7, 8 ], [ 2, 3, 4, 5, 10 ], [ 1, 3, 4, 6, 9 ], [ 1, 3, 6, 7, 9 ] ]
gap> Size(Orbit(g,[1,2,3,5,10],OnSets));
60
gap> Orbit(g,[1,2,3,7,9],OnSets);
[ [ 1, 2, 3, 7, 9 ], [ 1, 4, 5, 6, 9 ], [ 3, 4, 6, 7, 9 ], [ 1, 2, 4, 6, 10 ], [ 2, 3, 6, 9, 10 ],
  [ 1, 3, 5, 7, 10 ], [ 2, 4, 7, 8, 10 ], [ 3, 5, 6, 9, 10 ], [ 2, 6, 7, 8, 9 ], [ 1, 3, 4, 8, 9 ],
  [ 4, 5, 7, 8, 10 ], [ 3, 5, 7, 8, 10 ], [ 1, 2, 6, 8, 9 ], [ 3, 4, 5, 8, 9 ], [ 2, 3, 5, 6, 7 ],
  [ 1, 6, 7, 8, 9 ], [ 4, 5, 6, 9, 10 ], [ 1, 3, 5, 8, 10 ], [ 2, 4, 6, 8, 10 ], [ 1, 3, 5, 8, 9 ],
  [ 2, 4, 5, 6, 7 ], [ 1, 5, 6, 9, 10 ], [ 2, 3, 4, 5, 6 ], [ 2, 3, 7, 9, 10 ], [ 1, 2, 6, 8, 10 ],
  [ 1, 4, 5, 8, 9 ], [ 2, 5, 7, 8, 10 ], [ 2, 3, 4, 5, 7 ], [ 1, 2, 3, 7, 10 ], [ 1, 2, 3, 9, 10 ],
  [ 1, 2, 4, 6, 8 ], [ 1, 2, 7, 9, 10 ], [ 3, 6, 7, 8, 9 ], [ 2, 4, 5, 7, 8 ], [ 1, 2, 7, 8, 9 ],
  [ 1, 4, 5, 6, 10 ], [ 2, 4, 5, 8, 10 ], [ 1, 3, 5, 7, 8 ], [ 1, 3, 7, 9, 10 ], [ 3, 4, 7, 8, 9 ],
  [ 4, 6, 7, 8, 9 ], [ 2, 4, 5, 7, 10 ], [ 1, 4, 5, 9, 10 ], [ 3, 4, 6, 7, 8 ], [ 3, 4, 6, 8, 9 ],
  [ 2, 3, 5, 6, 9 ], [ 3, 4, 5, 6, 7 ], [ 1, 2, 4, 8, 10 ], [ 1, 5, 7, 8, 10 ], [ 1, 3, 4, 5, 9 ],
  [ 1, 4, 6, 9, 10 ], [ 2, 3, 5, 6, 10 ], [ 1, 3, 7, 8, 10 ], [ 1, 4, 6, 8, 10 ], [ 2, 3, 5, 9, 10 ],
  [ 2, 5, 6, 9, 10 ], [ 1, 2, 6, 7, 9 ], [ 2, 3, 4, 6, 7 ], [ 1, 3, 4, 5, 8 ], [ 1, 2, 6, 7, 8 ] ]
gap> Size(Orbit(g,[1,2,3,7,9],OnSets));
60
gap> Orbit(g,[1,2,6,9,10],OnSets);
[ [ 1, 2, 6, 9, 10 ], [ 1, 3, 5, 9, 10 ], [ 2, 4, 6, 7, 8 ], [ 1, 4, 6, 8, 9 ], [ 3, 4, 5, 7, 8 ],
  [ 2, 3, 5, 7, 10 ], [ 1, 3, 7, 8, 9 ], [ 2, 4, 5, 6, 10 ], [ 3, 4, 5, 6, 9 ], [ 1, 2, 7, 8, 10 ],
  [ 2, 3, 6, 7, 9 ], [ 1, 4, 5, 8, 10 ] ]
gap> Size(Orbit(g,[1,2,6,9,10],OnSets));
12
```

In[105]:= **Binomial[10, 5]**

Out[105]= 252

In[127]:= **Binomial[10, 5] == (30 + 30 + 60 + 60 + 60 + 12)**

Out[127]= True

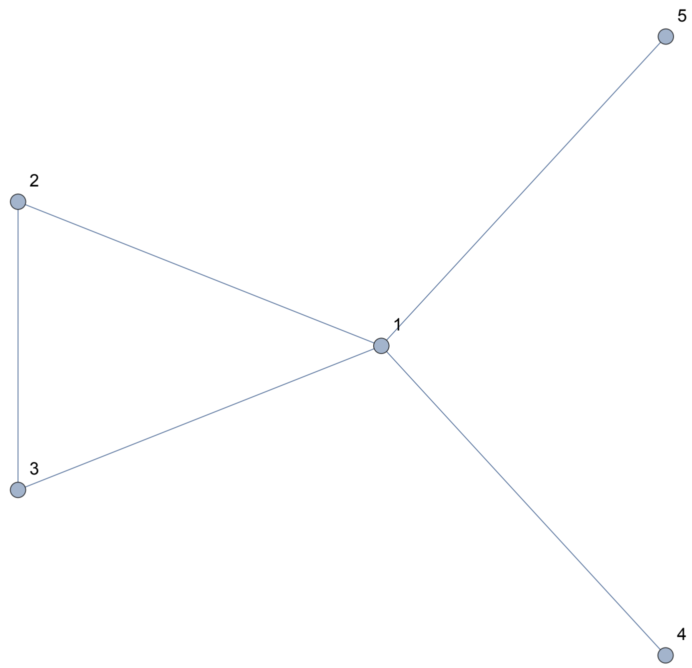
```
In[1281]:= L15 = {1, 1, 1, 1, 1, 0, 0, 0, 0, 0}

Out[1281]= {1, 1, 1, 1, 1, 0, 0, 0, 0, 0}

In[1282]:= A15 = {{0, 1, 1, 1, 1}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}}

Out[1282]= {{0, 1, 1, 1, 1}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {1, 0, 0, 0, 0}}

In[108]:= MatrixForm[A15]
```

Out[108]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[1283]:= G15 = AdjacencyGraph[A15, VertexLabels → "Name"]
```

Out[1283]=



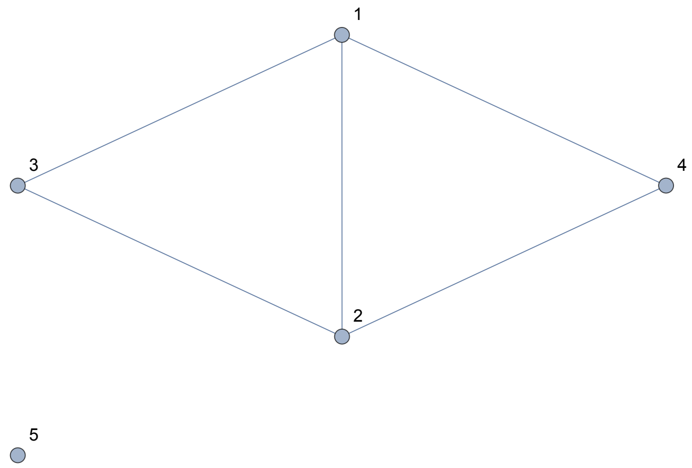

```
In[1284]:= L16 = {1, 1, 1, 0, 1, 1, 0, 0, 0, 0}

Out[1284]= {1, 1, 1, 0, 1, 1, 0, 0, 0, 0}

In[1285]:= A16 = {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {0, 0, 0, 0, 0}}

Out[1285]= {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[112]:= MatrixForm[A16]
```

Out[112]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1286]:=
```
G16 = AdjacencyGraph[A16, VertexLabels → "Name"]
```

Out[1286]=



In[1287]:=
```
L17 = {1, 1, 1, 0, 1, 0, 1, 0, 0, 0}
```

Out[1287]= {1, 1, 1, 0, 1, 0, 1, 0, 0, 0}

In[1288]:=
```
A17 = {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 1}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}}
```

Out[1288]= {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 1}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}}

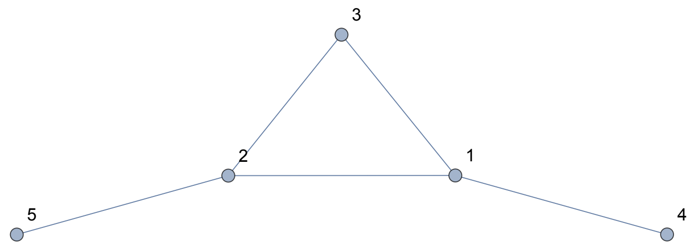
In[116]:=
```
MatrixForm[A17]
```

Out[116]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1289]:=
```
G17 = AdjacencyGraph[A17, VertexLabels → "Name"]
```

Out[1289]=



In[1290]:=
```
L18 = {1, 1, 1, 0, 1, 0, 0, 0, 0, 1}
```

Out[1290]= {1, 1, 1, 0, 1, 0, 0, 0, 0, 1}

In[1291]:=
```
A18 = {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 1}, {0, 0, 0, 1, 0}}
```

Out[1291]= {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 1}, {0, 0, 0, 1, 0}}

In[120]:=
```
MatrixForm[A18]
```

Out[120]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[1292]:= **G18 = AdjacencyGraph[A18, VertexLabels → "Name"]**

Out[1292]=

In[1293]:= **L19 = {1, 1, 1, 0, 0, 0, 1, 0, 1, 0}**

Out[1293]= {1, 1, 1, 0, 0, 0, 1, 0, 1, 0}

In[1294]:= **A19 = {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 0}, {0, 1, 1, 0, 0}}**

Out[1294]= {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 0}, {0, 1, 1, 0, 0}}

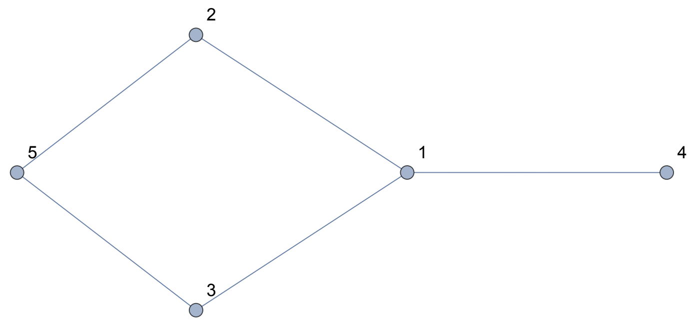
In[125]:= **MatrixForm[A19]**

Out[125]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

In[1295]:= **G19 = AdjacencyGraph[A19, VertexLabels → "Name"]**

Out[1295]=



In[1296]:= **L20 = {1, 1, 0, 0, 0, 1, 0, 0, 1, 1}**

Out[1296]= {1, 1, 0, 0, 0, 1, 0, 0, 1, 1}

In[1297]:= **A20 = {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 0}, {1, 0, 0, 0, 1}, {0, 1, 0, 0, 1}, {0, 0, 1, 1, 0}}**

Out[1297]= {{0, 1, 1, 0, 0}, {1, 0, 0, 1, 0}, {1, 0, 0, 0, 1}, {0, 1, 0, 0, 1}, {0, 0, 1, 1, 0}}

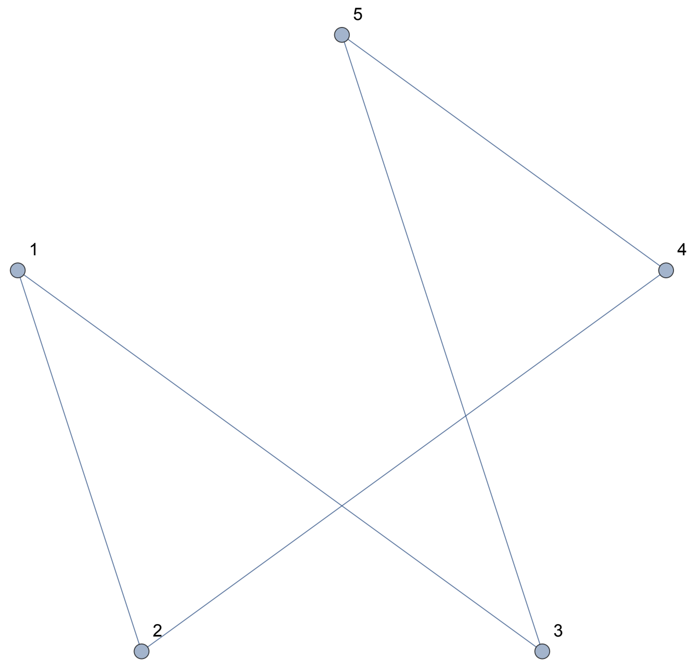
In[131]:= **MatrixForm[A20]**

Out[131]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

In[1298]:=
```
G20 = AdjacencyGraph[A20, VertexLabels → "Name"]
```

Out[1298]=




```
gap> Orbit(g,[1,2,3,4,5,6],OnSets);
[ [ 1, 2, 3, 4, 5, 6 ], [ 1, 2, 3, 5, 6, 7 ], [ 1, 2, 4, 7, 9, 10 ], [ 1, 2, 3, 4, 7, 9 ], [ 1, 4, 5, 7, 9, 10 ],
  [ 3, 4, 6, 7, 8, 10 ], [ 1, 4, 5, 6, 7, 9 ], [ 3, 4, 6, 7, 9, 10 ], [ 1, 3, 4, 6, 8, 10 ], [ 2, 3, 5, 8, 9, 10 ],
  [ 1, 2, 3, 4, 6, 10 ], [ 2, 3, 6, 8, 9, 10 ], [ 1, 3, 6, 7, 8, 10 ], [ 2, 4, 5, 8, 9, 10 ], [ 2, 5, 6, 8, 9, 10 ],
  [ 1, 5, 6, 7, 8, 9 ], [ 1, 3, 5, 6, 7, 10 ], [ 2, 4, 7, 8, 9, 10 ], [ 3, 5, 6, 8, 9, 10 ], [ 2, 5, 6, 7, 8, 9 ],
  [ 2, 3, 4, 5, 8, 9 ], [ 2, 5, 7, 8, 9, 10 ], [ 1, 5, 6, 7, 8, 10 ], [ 1, 2, 5, 6, 7, 8 ], [ 1, 2, 3, 4, 8, 9 ],
  [ 4, 5, 7, 8, 9, 10 ], [ 3, 5, 6, 7, 8, 10 ], [ 1, 2, 5, 6, 8, 9 ], [ 1, 5, 6, 7, 9, 10 ], [ 1, 3, 5, 6, 7, 8 ],
  [ 1, 2, 3, 4, 8, 10 ], [ 1, 2, 3, 4, 5, 8 ], [ 1, 2, 3, 4, 5, 7 ], [ 4, 5, 6, 7, 9, 10 ], [ 1, 3, 5, 6, 8, 10 ],
  [ 2, 3, 4, 6, 8, 10 ], [ 1, 2, 3, 5, 8, 9 ], [ 1, 2, 4, 5, 6, 7 ], [ 1, 2, 3, 4, 9, 10 ], [ 1, 2, 3, 4, 6, 8 ],
  [ 1, 2, 3, 4, 5, 9 ], [ 1, 3, 4, 7, 9, 10 ], [ 2, 3, 4, 7, 9, 10 ], [ 1, 2, 3, 6, 8, 10 ], [ 1, 2, 4, 5, 8, 9 ],
  [ 1, 2, 3, 4, 7, 10 ], [ 4, 6, 7, 8, 9, 10 ], [ 2, 4, 5, 7, 9, 10 ], [ 1, 2, 5, 6, 7, 9 ], [ 1, 4, 6, 7, 9, 10 ],
  [ 3, 4, 6, 8, 9, 10 ], [ 3, 6, 7, 8, 9, 10 ], [ 2, 4, 5, 7, 8, 9 ], [ 1, 2, 5, 7, 8, 9 ], [ 1, 4, 5, 6, 7, 10 ],
  [ 3, 4, 7, 8, 9, 10 ], [ 2, 3, 5, 6, 8, 10 ], [ 1, 2, 3, 4, 6, 7 ], [ 2, 3, 5, 6, 8, 9 ], [ 1, 3, 4, 5, 6, 7 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,6],OnSets));
60
gap> Orbit(g,[1,2,3,4,5,10],OnSets);
[ [ 1, 2, 3, 4, 5, 10 ], [ 1, 2, 5, 6, 7, 10 ], [ 1, 4, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 7, 8 ], [ 3, 4, 5, 6, 8, 10 ],
  [ 1, 4, 5, 6, 7, 8 ], [ 3, 4, 5, 7, 9, 10 ], [ 2, 3, 6, 7, 8, 10 ], [ 1, 2, 5, 8, 9, 10 ], [ 2, 4, 6, 7, 9, 10 ],
  [ 1, 3, 6, 8, 9, 10 ], [ 2, 3, 5, 7, 8, 9 ], [ 2, 4, 5, 6, 8, 9 ], [ 1, 3, 5, 6, 7, 9 ], [ 1, 2, 3, 4, 6, 9 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,10],OnSets));
15
gap> Orbit(g,[1,2,3,5,6,8],OnSets);
[ [ 1, 2, 3, 5, 6, 8 ], [ 1, 2, 4, 5, 7, 9 ], [ 1, 3, 4, 6, 7, 10 ], [ 2, 3, 4, 8, 9, 10 ], [ 5, 6, 7, 8, 9, 10 ] ]
gap> Size(Orbit(g,[1,2,3,5,6,8],OnSets));
5
gap> Orbit(g,[1,2,3,5,6,9],OnSets);
[ [ 1, 2, 3, 5, 6, 9 ], [ 1, 2, 4, 6, 7, 9 ], [ 1, 3, 4, 5, 7, 9 ], [ 2, 3, 4, 6, 7, 10 ], [ 1, 3, 4, 6, 9, 10 ],
  [ 3, 4, 5, 6, 7, 10 ], [ 2, 3, 7, 8, 9, 10 ], [ 1, 3, 6, 7, 9, 10 ], [ 2, 4, 6, 8, 9, 10 ],
  [ 1, 2, 3, 8, 9, 10 ], [ 4, 5, 6, 8, 9, 10 ], [ 3, 5, 6, 7, 8, 9 ], [ 2, 3, 4, 6, 8, 9 ], [ 3, 5, 7, 8, 9, 10 ],
  [ 2, 5, 6, 7, 8, 10 ], [ 1, 5, 6, 8, 9, 10 ], [ 4, 5, 6, 7, 8, 9 ], [ 1, 2, 5, 6, 8, 10 ], [ 2, 5, 6, 7, 9, 10 ],
  [ 1, 3, 5, 6, 8, 9 ], [ 2, 3, 4, 5, 8, 10 ], [ 1, 2, 3, 5, 7, 8 ], [ 1, 2, 4, 5, 6, 8 ], [ 2, 3, 4, 7, 8, 9 ],
```

```
 [ 1, 2, 3, 5, 6, 10 ], [ 1, 2, 3, 5, 8, 10 ], [ 1, 2, 4, 5, 7, 8 ], [ 2, 3, 4, 5, 9, 10 ], [ 1, 2, 3, 6, 7, 8 ],
 [ 1, 2, 3, 6, 8, 9 ], [ 1, 2, 4, 5, 6, 9 ], [ 1, 5, 7, 8, 9, 10 ], [ 1, 2, 4, 5, 7, 10 ], [ 3, 5, 6, 7, 9, 10 ],
 [ 1, 2, 4, 7, 8, 9 ], [ 1, 4, 5, 7, 8, 9 ], [ 1, 3, 4, 5, 7, 10 ], [ 1, 6, 7, 8, 9, 10 ], [ 1, 3, 4, 5, 6, 8 ],
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 [ 1, 4, 6, 7, 8, 10 ], [ 3, 4, 5, 8, 9, 10 ], [ 2, 6, 7, 8, 9, 10 ], [ 2, 3, 5, 6, 7, 8 ], [ 1, 2, 5, 7, 9, 10 ],
 [ 1, 3, 4, 6, 7, 8 ] ]
gap> Size(Orbit(g,[1,2,3,5,6,9],OnSets));
60
gap> Orbit(g,[1,2,3,5,7,10],OnSets);
[ [ 1, 2, 3, 5, 7, 10 ], [ 1, 2, 4, 5, 6, 10 ], [ 1, 3, 4, 7, 8, 9 ], [ 1, 2, 4, 7, 8, 10 ], [ 1, 4, 6, 7, 8, 9 ],
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 [ 2, 3, 6, 7, 9, 10 ], [ 1, 2, 6, 8, 9, 10 ], [ 1, 3, 4, 5, 8, 10 ], [ 2, 4, 6, 7, 8, 10 ],
 [ 1, 3, 5, 8, 9, 10 ], [ 2, 3, 5, 6, 7, 10 ], [ 1, 2, 7, 8, 9, 10 ], [ 2, 3, 6, 7, 8, 9 ], [ 2, 4, 5, 6, 7, 8 ],
 [ 1, 2, 6, 7, 8, 10 ], [ 1, 4, 5, 8, 9, 10 ], [ 2, 3, 5, 7, 8, 10 ], [ 1, 4, 5, 6, 8, 9 ], [ 1, 2, 3, 7, 8, 9 ],
 [ 3, 4, 5, 6, 7, 8 ], [ 3, 4, 5, 6, 8, 9 ], [ 2, 3, 5, 6, 7, 9 ], [ 2, 3, 4, 5, 7, 8 ], [ 1, 4, 5, 6, 8, 10 ],
 [ 2, 4, 5, 6, 8, 10 ], [ 1, 3, 5, 7, 8, 9 ], [ 3, 4, 5, 6, 7, 9 ], [ 2, 3, 4, 6, 7, 8 ], [ 1, 2, 3, 5, 9, 10 ],
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 [ 1, 2, 4, 5, 8, 10 ], [ 1, 2, 5, 7, 8, 10 ], [ 1, 2, 4, 6, 8, 9 ], [ 1, 3, 4, 5, 6, 9 ], [ 2, 3, 4, 6, 7, 9 ],
 [ 1, 2, 3, 6, 9, 10 ], [ 2, 3, 5, 7, 9, 10 ], [ 1, 2, 4, 6, 7, 8 ], [ 1, 2, 6, 7, 9, 10 ], [ 1, 2, 4, 6, 9, 10 ],
 [ 1, 3, 4, 6, 8, 9 ], [ 3, 4, 5, 7, 8, 9 ], [ 1, 3, 4, 5, 7, 8 ], [ 2, 4, 5, 6, 7, 10 ], [ 1, 3, 5, 6, 9, 10 ],
 [ 2, 4, 6, 7, 8, 9 ], [ 2, 4, 5, 6, 9, 10 ], [ 1, 3, 6, 7, 8, 9 ], [ 2, 3, 4, 5, 7, 10 ], [ 1, 3, 4, 5, 9, 10 ],
 [ 1, 3, 7, 8, 9, 10 ], [ 1, 4, 6, 8, 9, 10 ] ]
gap> Size(Orbit(g,[1,2,3,5,7,10],OnSets));
60
gap> Orbit(g,[1,2,3,7,9,10],OnSets);
[ [ 1, 2, 3, 7, 9, 10 ], [ 1, 4, 5, 6, 9, 10 ], [ 3, 4, 6, 7, 8, 9 ], [ 1, 2, 4, 6, 8, 10 ], [ 2, 3, 5, 6, 9, 10 ],
 [ 1, 3, 5, 7, 8, 10 ], [ 2, 4, 5, 7, 8, 10 ], [ 1, 2, 6, 7, 8, 9 ], [ 1, 3, 4, 5, 8, 9 ], [ 2, 3, 4, 5, 6, 7 ] ]
gap> Size(Orbit(g,[1,2,3,7,9,10],OnSets));
10
```

In[133]:= **Binomial[10, 6]**

Out[133]= 210

In[1299]:= **Binomial[10, 6] == (60 + 15 + 5 + 60 + 60 + 10)**

Out[1299]= True

In[1300]:= **L21 = {1, 1, 1, 1, 1, 1, 0, 0, 0, 0}**

Out[1300]= {1, 1, 1, 1, 1, 1, 0, 0, 0, 0}

In[1301]:= **A21 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}}**

Out[1301]= {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 0}}

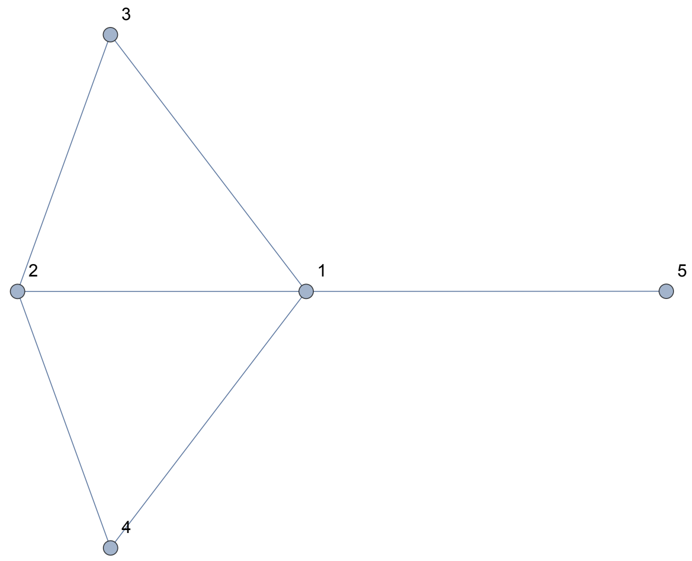
In[137]:= **MatrixForm[A21]**

Out[137]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1302]:= **G21 = AdjacencyGraph[A21, VertexLabels → "Name"]**

Out[1302]=



In[1303]:= **L22 = {1, 1, 1, 1, 1, 0, 0, 0, 0, 1}**

Out[1303]= {1, 1, 1, 1, 1, 0, 0, 0, 0, 1}

In[1304]:= **A22 = {{0, 1, 1, 1, 1}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 1, 0}}**

Out[1304]= {{0, 1, 1, 1, 1}, {1, 0, 1, 0, 0}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 1, 0}}

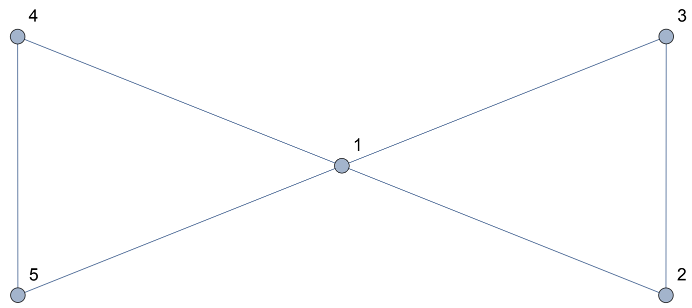
In[141]:= **MatrixForm[A22]**

Out[141]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[1305]:= **G22 = AdjacencyGraph[A22, VertexLabels → "Name"]**

Out[1305]=



In[1306]:= **L23 = {1, 1, 1, 0, 1, 1, 0, 1, 0, 0}**

Out[1306]= {1, 1, 1, 0, 1, 1, 0, 1, 0, 0}

In[1307]:= **A23 = {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 1, 0}, {1, 1, 1, 0, 0}, {0, 0, 0, 0, 0}}**

Out[1307]= {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 1, 0}, {1, 1, 1, 0, 0}, {0, 0, 0, 0, 0}}

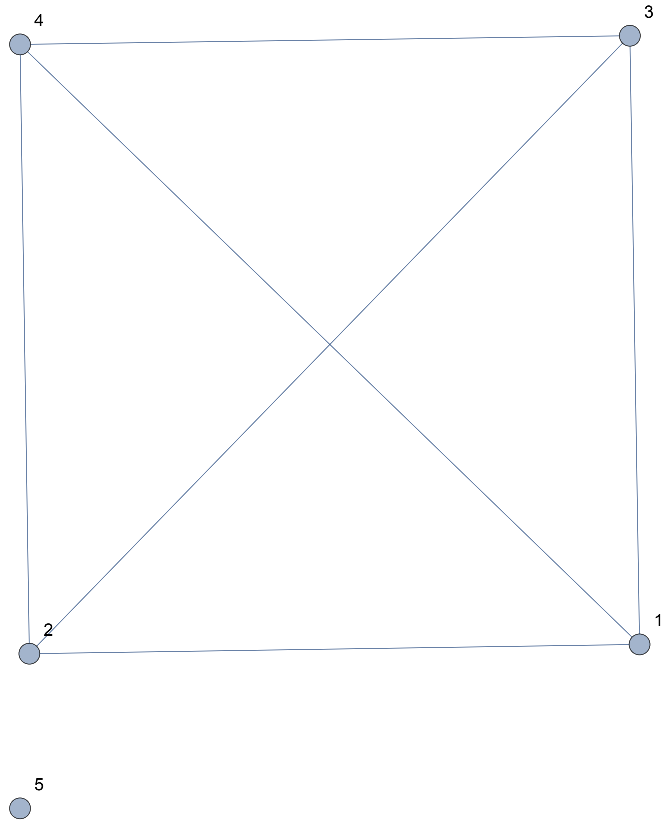
In[145]:= `MatrixForm[A23]`

Out[145]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1308]:= `G23 = AdjacencyGraph[A23, VertexLabels → "Name"]`

Out[1308]=



In[1309]:= `L24 = {1, 1, 1, 0, 1, 1, 0, 0, 1, 0}`

Out[1309]= {1, 1, 1, 0, 1, 1, 0, 0, 1, 0}

In[1310]:= `A24 = {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 0}, {0, 0, 1, 0, 0}}`

Out[1310]= {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 0}, {0, 0, 1, 0, 0}}

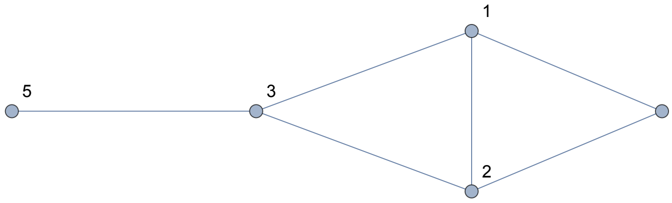
In[149]:= `MatrixForm[A24]`

Out[149]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

In[1311]:= `G24 = AdjacencyGraph[A24, VertexLabels → "Name"]`

Out[1311]=



In[1312]:= `L25 = {1, 1, 1, 0, 1, 0, 1, 0, 0, 1}`

Out[1312]= {1, 1, 1, 0, 1, 0, 1, 0, 0, 1}

In[1313]:= `A25 = {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 1}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 1}, {0, 1, 0, 1, 0}}`

Out[1313]= {{0, 1, 1, 1, 0}, {1, 0, 1, 0, 1}, {1, 1, 0, 0, 0}, {1, 0, 0, 0, 1}, {0, 1, 0, 1, 0}}

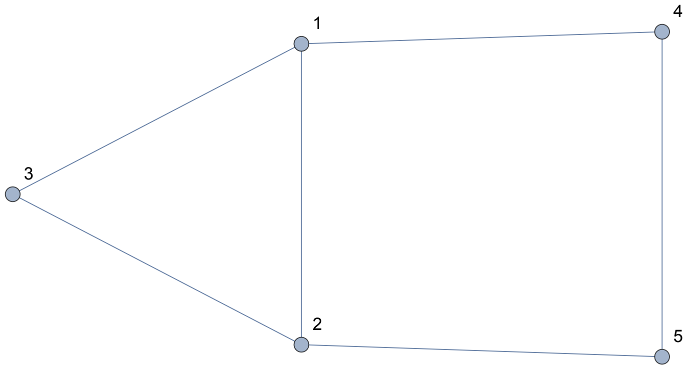
In[153]:= `MatrixForm[A25]`

Out[153]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

In[1314]:= `G25 = AdjacencyGraph[A25, VertexLabels → "Name"]`

Out[1314]=



In[1315]:= `L26 = {1, 1, 1, 0, 0, 0, 1, 0, 1, 1}`

Out[1315]= {1, 1, 1, 0, 0, 0, 1, 0, 1, 1}

In[1317]:= `A26 = {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 1}, {0, 1, 1, 1, 0}}`

Out[1317]= {{0, 1, 1, 1, 0}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 1}, {1, 0, 0, 0, 1}, {0, 1, 1, 1, 0}}

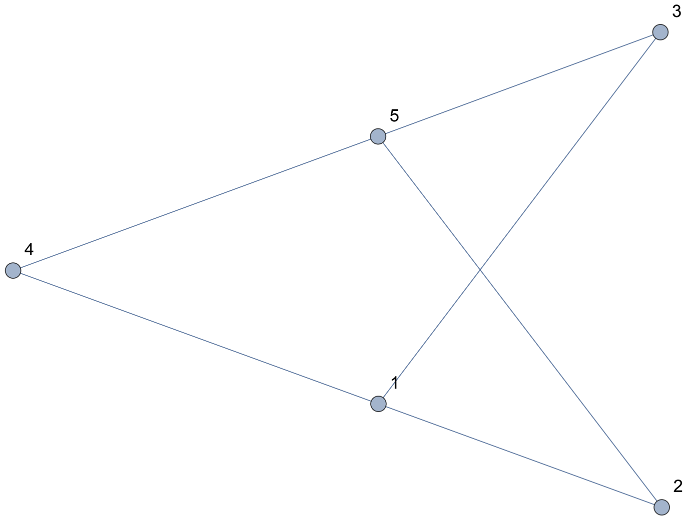
In[158]:= `MatrixForm[A26]`

Out[158]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

In[1318]:= **G26 = AdjacencyGraph[A26, VertexLabels → "Name"]**

Out[1318]=




```
gap> Orbit(g,[1,2,3,4,5,6,7],OnSets);
[ [ 1, 2, 3, 4, 5, 6, 7 ], [ 1, 2, 3, 4, 7, 9, 10 ], [ 1, 4, 5, 6, 7, 9, 10 ], [ 3, 4, 6, 7, 8, 9, 10 ],
  [ 1, 2, 3, 4, 6, 8, 10 ], [ 2, 3, 5, 6, 8, 9, 10 ], [ 1, 3, 5, 6, 7, 8, 10 ], [ 2, 4, 5, 7, 8, 9, 10 ],
  [ 1, 2, 5, 6, 7, 8, 9 ], [ 1, 2, 3, 4, 5, 8, 9 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,6,7],OnSets));
10
gap> Orbit(g,[1,2,3,4,5,6,8],OnSets);
[ [ 1, 2, 3, 4, 5, 6, 8 ], [ 1, 2, 3, 5, 6, 7, 8 ], [ 1, 2, 4, 5, 7, 9, 10 ], [ 1, 2, 3, 4, 5, 7, 9 ],
  [ 1, 3, 4, 6, 7, 8, 10 ], [ 1, 2, 4, 5, 6, 7, 9 ], [ 1, 3, 4, 6, 7, 9, 10 ], [ 2, 3, 4, 5, 8, 9, 10 ],
  [ 1, 2, 3, 4, 6, 7, 10 ], [ 2, 3, 4, 6, 8, 9, 10 ], [ 2, 5, 6, 7, 8, 9, 10 ], [ 1, 5, 6, 7, 8, 9, 10 ],
  [ 1, 3, 4, 5, 6, 7, 10 ], [ 2, 3, 4, 7, 8, 9, 10 ], [ 3, 5, 6, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 8, 9, 10 ],
  [ 4, 5, 6, 7, 8, 9, 10 ], [ 1, 2, 3, 5, 6, 8, 9 ], [ 1, 2, 3, 5, 6, 8, 10 ], [ 1, 2, 4, 5, 7, 8, 9 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,6,8],OnSets));
20
gap> Orbit(g,[1,2,3,4,5,6,9],OnSets);
[ [ 1, 2, 3, 4, 5, 6, 9 ], [ 1, 2, 3, 5, 6, 7, 9 ], [ 1, 2, 4, 6, 7, 9, 10 ], [ 1, 2, 3, 4, 6, 7, 9 ],
  [ 1, 3, 4, 5, 7, 9, 10 ], [ 2, 3, 4, 6, 7, 8, 10 ], [ 1, 3, 4, 5, 6, 7, 9 ], [ 2, 3, 4, 6, 7, 9, 10 ],
  [ 1, 3, 4, 6, 8, 9, 10 ], [ 3, 4, 5, 6, 7, 8, 10 ], [ 2, 3, 5, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 6, 9, 10 ],
  [ 3, 4, 5, 6, 7, 9, 10 ], [ 2, 3, 6, 7, 8, 9, 10 ], [ 1, 3, 6, 7, 8, 9, 10 ], [ 2, 4, 5, 6, 8, 9, 10 ],
  [ 1, 2, 3, 5, 8, 9, 10 ], [ 1, 3, 5, 6, 7, 8, 9 ], [ 1, 3, 5, 6, 7, 9, 10 ], [ 2, 4, 6, 7, 8, 9, 10 ],
  [ 1, 2, 3, 6, 8, 9, 10 ], [ 3, 4, 5, 6, 8, 9, 10 ], [ 2, 3, 5, 6, 7, 8, 9 ], [ 2, 3, 4, 5, 6, 8, 9 ],
  [ 1, 2, 5, 6, 7, 8, 10 ], [ 1, 2, 5, 6, 8, 9, 10 ], [ 1, 4, 5, 6, 7, 8, 9 ], [ 1, 2, 3, 4, 6, 8, 9 ],
  [ 3, 4, 5, 7, 8, 9, 10 ], [ 2, 3, 5, 6, 7, 8, 10 ], [ 1, 3, 5, 6, 8, 9, 10 ], [ 2, 4, 5, 6, 7, 8, 9 ],
  [ 1, 2, 5, 6, 7, 9, 10 ], [ 1, 2, 3, 4, 5, 8, 10 ], [ 1, 2, 3, 4, 5, 7, 8 ], [ 1, 2, 4, 5, 6, 7, 8 ],
  [ 1, 2, 3, 4, 7, 8, 9 ], [ 1, 2, 3, 4, 5, 6, 10 ], [ 2, 4, 5, 6, 7, 9, 10 ], [ 2, 3, 4, 5, 6, 8, 10 ],
  [ 1, 2, 3, 5, 7, 8, 9 ], [ 1, 2, 4, 5, 6, 8, 9 ], [ 2, 3, 4, 5, 7, 8, 9 ], [ 1, 2, 3, 5, 6, 7, 10 ],
  [ 1, 2, 3, 4, 5, 9, 10 ], [ 1, 2, 3, 4, 6, 7, 8 ], [ 1, 4, 5, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 5, 7, 10 ],
  [ 1, 2, 4, 7, 8, 9, 10 ], [ 2, 3, 4, 5, 7, 9, 10 ], [ 1, 2, 3, 6, 7, 8, 10 ], [ 1, 2, 5, 7, 8, 9, 10 ],
  [ 1, 2, 4, 5, 6, 7, 10 ], [ 1, 4, 6, 7, 8, 9, 10 ], [ 1, 3, 4, 5, 6, 7, 8 ], [ 1, 3, 4, 5, 6, 8, 10 ],
  [ 1, 3, 4, 7, 8, 9, 10 ], [ 1, 2, 4, 5, 8, 9, 10 ], [ 1, 2, 3, 4, 7, 8, 10 ], [ 1, 4, 5, 6, 7, 8, 10 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,6,9],OnSets));
60
gap> Orbit(g,[1,2,3,5,6,9,10],OnSets);
[ [ 1, 2, 3, 5, 6, 9, 10 ], [ 1, 2, 4, 6, 7, 8, 9 ], [ 1, 3, 4, 5, 7, 8, 9 ], [ 2, 3, 4, 5, 6, 7, 10 ],
  [ 1, 3, 4, 5, 6, 9, 10 ], [ 1, 2, 3, 7, 8, 9, 10 ], [ 1, 2, 3, 6, 7, 9, 10 ], [ 1, 2, 4, 6, 8, 9, 10 ],
```

[1, 4, 5, 6, 8, 9, 10], [3, 4, 5, 6, 7, 8, 9], [2, 3, 4, 6, 7, 8, 9], [1, 3, 5, 7, 8, 9, 10],
[2, 4, 5, 6, 7, 8, 10], [1, 2, 4, 5, 6, 8, 10], [2, 3, 5, 6, 7, 9, 10], [1, 3, 4, 5, 6, 8, 9],
[2, 3, 4, 5, 7, 8, 10], [1, 2, 3, 5, 7, 8, 10], [1, 2, 4, 5, 7, 8, 10], [2, 3, 4, 5, 6, 9, 10],
[1, 2, 3, 6, 7, 8, 9], [1, 2, 4, 5, 6, 9, 10], [1, 3, 4, 5, 7, 8, 10], [1, 2, 6, 7, 8, 9, 10],
[2, 3, 4, 5, 6, 7, 9], [1, 2, 3, 5, 7, 9, 10], [1, 2, 4, 6, 7, 8, 10], [1, 3, 4, 5, 8, 9, 10],
[2, 3, 4, 5, 6, 7, 8], [1, 3, 4, 6, 7, 8, 9]]
gap> Size(Orbit(g,[1,2,3,5,6,9,10],OnSets));
30

In[161]:= **Binomial[10, 7]**

Out[161]= 120

In[176]:= **Binomial[10, 7] == (10 + 20 + 60 + 30)**

Out[176]= True

In[1319]:= **L27 = {1, 1, 1, 1, 1, 1, 1, 0, 0, 0}**

Out[1319]= {1, 1, 1, 1, 1, 1, 1, 0, 0, 0}

In[1320]:= **A27 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}}**

Out[1320]= {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 0, 0, 0}}

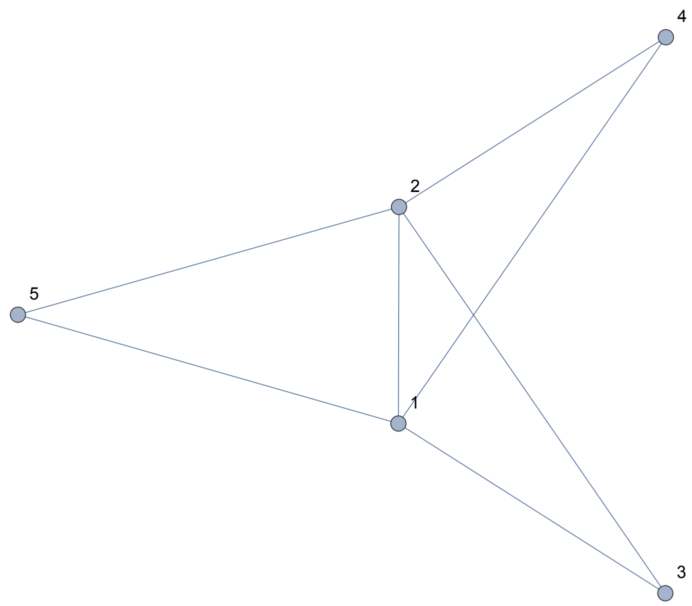
In[164]:= **MatrixForm[A27]**

Out[164]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1321]:= **G27 = AdjacencyGraph[A27, VertexLabels → "Name"]**

Out[1321]=



In[1322]:= **L28 = {1, 1, 1, 1, 1, 1, 0, 1, 0, 0}**

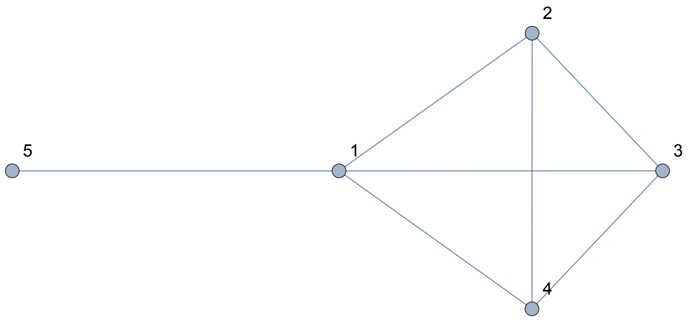
Out[1322]= {1, 1, 1, 1, 1, 1, 0, 1, 0, 0}

In[1323]:=
```
A28 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 1, 0}, {1, 1, 1, 0, 0}, {1, 0, 0, 0, 0}}
```
Out[1323]=
```
{{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 1, 0}, {1, 1, 1, 0, 0}, {1, 0, 0, 0, 0}}
```

In[168]:=
```
MatrixForm[A28]
```
Out[168]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1324]:=
```
G28 = AdjacencyGraph[A28, VertexLabels → "Name"]
```
Out[1324]=



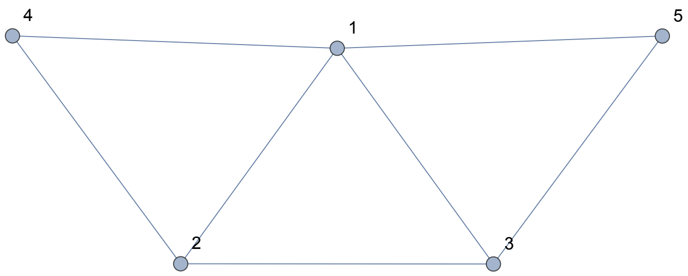
In[1325]:=
```
L29 = {1, 1, 1, 1, 1, 1, 0, 0, 1, 0}
```
Out[1325]=
```
{1, 1, 1, 1, 1, 1, 0, 0, 1, 0}
```

In[1326]:=
```
A29 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 0}, {1, 0, 1, 0, 0}}
```
Out[1326]=
```
{{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 0}, {1, 0, 1, 0, 0}}
```

In[173]:=
```
MatrixForm[A29]
```
Out[173]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

In[1327]:=
```
G29 = AdjacencyGraph[A29, VertexLabels → "Name"]
```
Out[1327]=



In[1328]:=
```
L30 = {1, 1, 1, 0, 1, 1, 0, 0, 1, 1}
```
Out[1328]=
```
{1, 1, 1, 0, 1, 1, 0, 0, 1, 1}
```

In[1329]:=
```
A30 = {{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 1}, {0, 0, 1, 1, 0}}
```
Out[1329]=
```
{{0, 1, 1, 1, 0}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 1}, {0, 0, 1, 1, 0}}
```

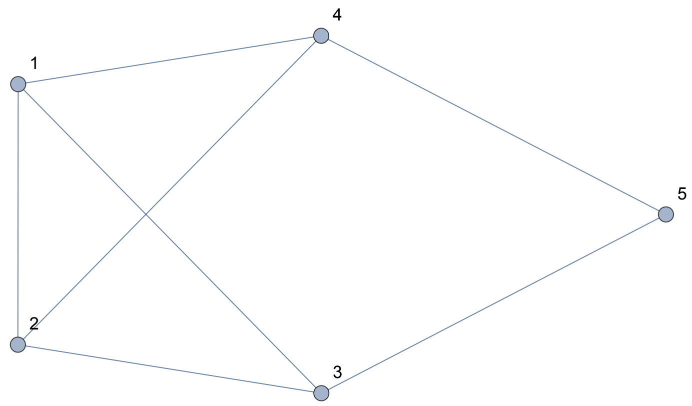
In[179]:= `MatrixForm[A30]`

Out[179]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

In[1330]:= `G30 = AdjacencyGraph[A30, VertexLabels → "Name"]`

Out[1330]=




```
gap> Orbit(g,[1,2,3,4,5,6,7,8],OnSets);
[ [ 1, 2, 3, 4, 5, 6, 7, 8 ], [ 1, 2, 3, 4, 5, 7, 9, 10 ], [ 1, 2, 4, 5, 6, 7, 9, 10 ],
  [ 1, 3, 4, 6, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 6, 7, 8, 10 ], [ 2, 3, 4, 5, 6, 8, 9, 10 ],
  [ 1, 3, 4, 5, 6, 7, 8, 10 ], [ 2, 3, 4, 5, 7, 8, 9, 10 ], [ 2, 3, 5, 6, 7, 8, 9, 10 ],
  [ 1, 2, 5, 6, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 5, 8, 9, 10 ], [ 2, 4, 5, 6, 7, 8, 9, 10 ],
  [ 1, 3, 5, 6, 7, 8, 9, 10 ], [ 1, 2, 3, 5, 6, 7, 8, 9 ], [ 1, 4, 5, 6, 7, 8, 9, 10 ],
  [ 1, 2, 3, 5, 6, 7, 8, 10 ], [ 1, 2, 3, 4, 6, 8, 9, 10 ], [ 1, 2, 3, 4, 5, 6, 8, 9 ],
  [ 1, 2, 3, 4, 5, 6, 7, 9 ], [ 1, 2, 3, 4, 7, 8, 9, 10 ], [ 1, 2, 3, 4, 5, 6, 8, 10 ],
  [ 1, 2, 3, 4, 5, 7, 8, 9 ], [ 1, 2, 3, 4, 6, 7, 9, 10 ], [ 3, 4, 5, 6, 7, 8, 9, 10 ],
  [ 1, 2, 4, 5, 7, 8, 9, 10 ], [ 1, 2, 4, 5, 6, 7, 8, 9 ], [ 1, 3, 4, 5, 6, 7, 9, 10 ],
  [ 2, 3, 4, 6, 7, 8, 9, 10 ], [ 1, 2, 3, 5, 6, 8, 9, 10 ], [ 1, 2, 3, 4, 5, 6, 7, 10 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,6,7,8],OnSets));
30
gap> Orbit(g,[1,2,3,4,5,6,9,10],OnSets);
[ [ 1, 2, 3, 4, 5, 6, 9, 10 ], [ 1, 2, 3, 5, 6, 7, 9, 10 ], [ 1, 2, 4, 6, 7, 8, 9, 10 ],
  [ 1, 2, 3, 4, 6, 7, 8, 9 ], [ 1, 3, 4, 5, 7, 8, 9, 10 ], [ 2, 3, 4, 5, 6, 7, 8, 10 ],
  [ 1, 3, 4, 5, 6, 7, 8, 9 ], [ 2, 3, 4, 5, 6, 7, 9, 10 ], [ 1, 3, 4, 5, 6, 8, 9, 10 ],
  [ 1, 2, 3, 5, 7, 8, 9, 10 ], [ 1, 2, 3, 6, 7, 8, 9, 10 ], [ 1, 2, 4, 5, 6, 8, 9, 10 ],
  [ 2, 3, 4, 5, 6, 7, 8, 9 ], [ 1, 2, 4, 5, 6, 7, 8, 10 ], [ 1, 2, 3, 4, 5, 7, 8, 10 ] ]
gap> Size(Orbit(g,[1,2,3,4,5,6,9,10],OnSets));
15
```

In[181]:= `Binomial[10, 8]`

Out[181]= 45

In[186]:= `Binomial[10, 8] == (30 + 15)`

Out[186]= True

In[1331]:= `L31 = {1, 1, 1, 1, 1, 1, 1, 1, 0, 0}`

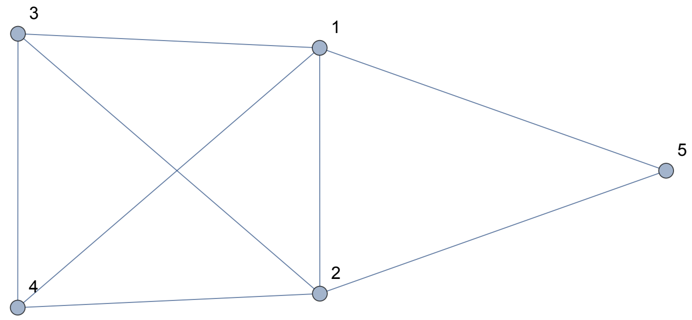
Out[1331]= {1, 1, 1, 1, 1, 1, 1, 1, 0, 0}

In[1332]:=
```
A31 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 1, 0}, {1, 1, 1, 0, 0}, {1, 1, 0, 0, 0}}
```
Out[1332]=
```
{{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 1, 0}, {1, 1, 1, 0, 0}, {1, 1, 0, 0, 0}}
```

In[184]:=
```
MatrixForm[A31]
```
Out[184]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1333]:=
```
G31 = AdjacencyGraph[A31, VertexLabels → "Name"]
```
Out[1333]=



In[1334]:=
```
L32 = {1, 1, 1, 1, 1, 1, 0, 0, 1, 1}
```
Out[1334]=
```
{1, 1, 1, 1, 1, 1, 0, 0, 1, 1}
```

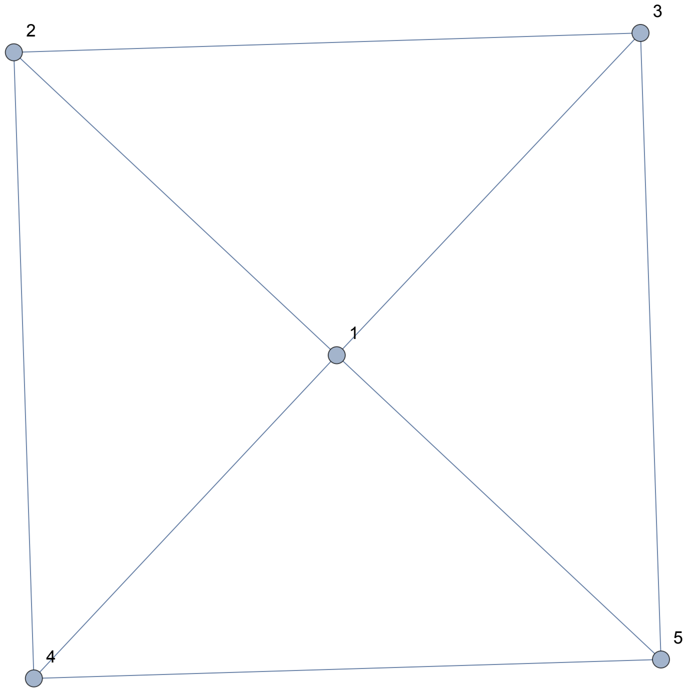
In[1335]:=
```
A32 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 1}, {1, 0, 1, 1, 0}}
```
Out[1335]=
```
{{0, 1, 1, 1, 1}, {1, 0, 1, 1, 0}, {1, 1, 0, 0, 1}, {1, 1, 0, 0, 1}, {1, 0, 1, 1, 0}}
```

In[189]:=
```
MatrixForm[A32]
```
Out[189]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

In[1336]:= **G32 = AdjacencyGraph[A32, VertexLabels → "Name"]**

Out[1336]=



gap> Orbit(g,[1,2,3,4,5,6,7,8,9],OnSets);
[[1, 2, 3, 4, 5, 6, 7, 8, 9], [1, 2, 3, 4, 5, 6, 7, 9, 10], [1, 2, 3, 4, 6, 7, 8, 9, 10],
[1, 3, 4, 5, 6, 7, 8, 9, 10], [2, 3, 4, 5, 6, 7, 8, 9, 10], [1, 2, 3, 4, 5, 6, 8, 9, 10],
[1, 2, 3, 5, 6, 7, 8, 9, 10], [1, 2, 4, 5, 6, 7, 8, 9, 10], [1, 2, 3, 4, 5, 7, 8, 9, 10],
[1, 2, 3, 4, 5, 6, 7, 8, 10]]
gap> Size(Orbit(g,[1,2,3,4,5,6,7,8,9],OnSets));
10

In[192]:= **Binomial[10, 9]**

Out[192]= 10

In[1337]:= **L33 = {1, 1, 1, 1, 1, 1, 1, 1, 1, 0}**

Out[1337]= {1, 1, 1, 1, 1, 1, 1, 1, 1, 0}

In[1338]:= **A33 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 0, 0}, {1, 1, 1, 0, 0}}**

Out[1338]= {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 0, 0}, {1, 1, 1, 0, 0}}

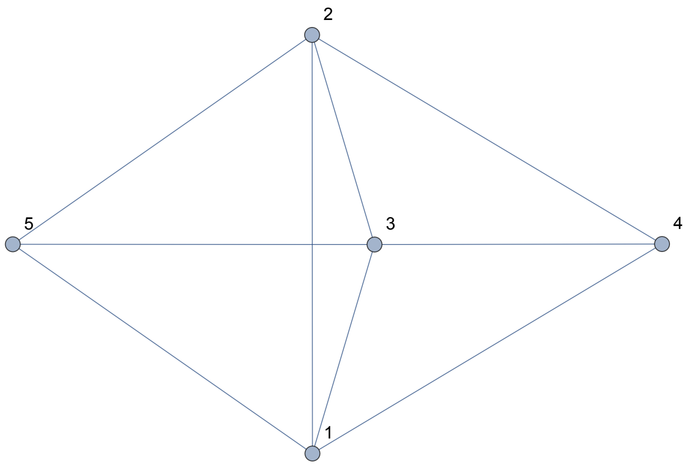
In[195]:= **MatrixForm[A33]**

Out[195]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

In[1339]:= **G33 = AdjacencyGraph[A33, VertexLabels → "Name"]**

Out[1339]=



gap> Orbit(g,[1,2,3,4,5,6,7,8,9,10],OnSets);
[[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]]
gap> Size(Orbit(g,[1,2,3,4,5,6,7,8,9,10],OnSets));
1

In[1350]:= **Binomial[10, 10]**

Out[1350]= 1

In[1340]:= **L34 = {1, 1, 1, 1, 1, 1, 1, 1, 1, 1}**

Out[1340]= {1, 1, 1, 1, 1, 1, 1, 1, 1, 1}

In[1349]:= **A34 = {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 0, 1}, {1, 1, 1, 1, 0}}**

Out[1349]= {{0, 1, 1, 1, 1}, {1, 0, 1, 1, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 0, 1}, {1, 1, 1, 1, 0}}

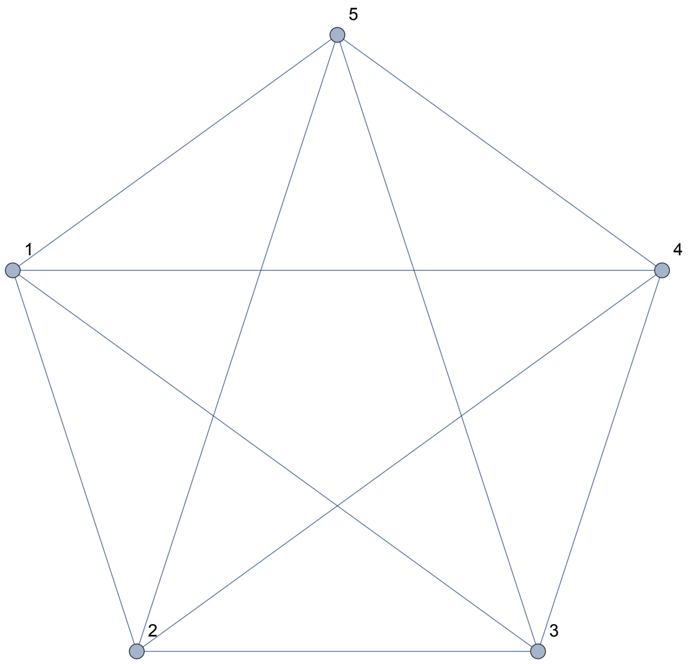
In[1348]:= **MatrixForm[A34]**

Out[1348]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

In[1342]:=
```
G34 = AdjacencyGraph[A34, VertexLabels → "Name"]
```

Out[1342]=



#Calcolare lo spettro della matrice di adiacenze (A1, A2, A3, ...) e quello della matrice di Laplace (L1, L2, L3, ...) per ognuno dei grafi (G1, G2, G3, ...) di cui al punto precedente.

Per ogni matrice di adiacenza (A1, A2, A3, ...) calcoliamo lo spettro, ovvero l'insieme dei suoi autovalori, con la funzione EigenValues. Di ogni grafo calcoliamo la rispettiva matrice dei gradi (D1, D2, D3, ...) con le funzioni VertexDegree e DiagonalMatrix, ne otteniamo la matrice di Laplace effettuando la sottrazione tra la matrice dei gradi e quella di adiacenza (L1=D1-A1, L2=D2-A2, L3=D3-A3, ...) e di quest'ultima ne calcoliamo lo spettro.

In[1346]:=
```
Eigenvalues[A1]
```

Out[1346]=
```
{0, 0, 0, 0, 0}
```

In[1344]:=
```
VertexDegree[G1]
```

Out[1344]=
```
{0, 0, 0, 0, 0}
```

In[1345]:=
```
D1 = DiagonalMatrix[%]
```

Out[1345]=
```
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

In[1347]:=
```
MatrixForm[D1]
```

Out[1347]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1351]:=
```
L1 = D1 - A1
```

Out[1351]=
```
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

In[1352]:= **MatrixForm[L1]**

Out[1352]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1353]:= **Eigenvalues[L1]**

Out[1353]= {0, 0, 0, 0, 0}

In[1354]:= **Eigenvalues[A2]**

Out[1354]= {-1, 1, 0, 0, 0}

In[1355]:= **VertexDegree[G2]**

Out[1355]= {1, 1, 0, 0, 0}

In[1356]:= **D2 = DiagonalMatrix[%]**

Out[1356]= {{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1357]:= **MatrixForm[D2]**

Out[1357]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1358]:= **L2 = D2 - A2**

Out[1358]= {{1, -1, 0, 0, 0}, {-1, 1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1359]:= **MatrixForm[L2]**

Out[1359]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1360]:= **Eigenvalues[L2]**

Out[1360]= {2, 0, 0, 0, 0}

In[1361]:= **Eigenvalues[A3]**

Out[1361]= $\left\{-\sqrt{2}, \sqrt{2}, 0, 0, 0\right\}$

In[1362]:= **VertexDegree[G3]**

Out[1362]= {2, 1, 1, 0, 0}

In[1363]:= **D3 = DiagonalMatrix[%]**

Out[1363]= {{2, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1364]:= `MatrixForm[D3]`

Out[1364]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1365]:= `L3 = D3 - A3`

Out[1365]= {{2, -1, -1, 0, 0}, {-1, 1, 0, 0, 0}, {-1, 0, 1, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1366]:= `MatrixForm[L3]`

Out[1366]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1367]:= `Eigenvalues[L3]`

Out[1367]= {3, 1, 0, 0, 0}

In[1368]:= `VertexDegree[G3]`

Out[1368]= {2, 1, 1, 0, 0}

In[1369]:= `D3 = DiagonalMatrix[%]`

Out[1369]= {{2, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1370]:= `MatrixForm[D3]`

Out[1370]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1371]:= `L3 = D3 - A3`

Out[1371]= {{2, -1, -1, 0, 0}, {-1, 1, 0, 0, 0}, {-1, 0, 1, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1372]:= `MatrixForm[L3]`

Out[1372]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1373]:= `Eigenvalues[L3]`

Out[1373]= {3, 1, 0, 0, 0}

In[1374]:= `Eigenvalues[A4]`

Out[1374]= {-1, -1, 1, 1, 0}

In[1375]:= **VertexDegree[G4]**

Out[1375]= {1, 1, 1, 1, 0}

In[1376]:= **D4 = DiagonalMatrix[%]**

Out[1376]= {{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1377]:= **MatrixForm[D4]**

Out[1377]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1378]:= **L4 = D4 - A4**

Out[1378]= {{1, -1, 0, 0, 0}, {-1, 1, 0, 0, 0}, {0, 0, 1, -1, 0}, {0, 0, -1, 1, 0}, {0, 0, 0, 0, 0}}

In[1379]:= **MatrixForm[L4]**

Out[1379]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1380]:= **Eigenvalues[L4]**

Out[1380]= {2, 2, 0, 0, 0}

In[1381]:= **Eigenvalues[A5]**

Out[1381]= $\left\{-\sqrt{3}, \sqrt{3}, 0, 0, 0\right\}$

In[1382]:= **VertexDegree[G5]**

Out[1382]= {3, 1, 1, 1, 0}

In[1383]:= **D5 = DiagonalMatrix[%]**

Out[1383]= {{3, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1384]:= **MatrixForm[D5]**

Out[1384]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1385]:= **L5 = D5 - A5**

Out[1385]= {{3, -1, -1, -1, 0}, {-1, 1, 0, 0, 0},
{-1, 0, 1, 0, 0}, {-1, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1386]:= `MatrixForm[L5]`

Out[1386]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1387]:= `Eigenvalues[L5]`

Out[1387]= {4, 1, 1, 0, 0}

In[1388]:= `Eigenvalues[A6]`

Out[1388]= {2, -1, -1, 0, 0}

In[1389]:= `VertexDegree[G6]`

Out[1389]= {2, 2, 2, 0, 0}

In[1390]:= `D6 = DiagonalMatrix[%]`

Out[1390]= {{2, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1391]:= `MatrixForm[D6]`

Out[1391]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1392]:= `L6 = D6 - A6`

Out[1392]= {{2, -1, -1, 0, 0}, {-1, 2, -1, 0, 0},
 {-1, -1, 2, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}

In[1393]:= `MatrixForm[L6]`

Out[1393]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1394]:= `Eigenvalues[L6]`

Out[1394]= {3, 3, 0, 0, 0}

In[1395]:= `Eigenvalues[A7]`

Out[1395]= $\left\{\frac{1}{2}\left(-1-\sqrt{5}\right), \frac{1}{2}\left(1+\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{5}\right), \frac{1}{2}\left(-1+\sqrt{5}\right), 0\right\}$

In[1396]:= `VertexDegree[G7]`

Out[1396]= {2, 2, 1, 1, 0}

In[1397]:= `D7 = DiagonalMatrix[%]`

Out[1397]= {{2, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1398]:= **MatrixForm[D7]**

Out[1398]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1399]:= **L7 = D7 - A7**

Out[1399]= {{2, -1, -1, 0, 0}, {-1, 2, 0, -1, 0},
{-1, 0, 1, 0, 0}, {0, -1, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1400]:= **MatrixForm[L7]**

Out[1400]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1401]:= **Eigenvalues[L7]**

Out[1401]= $\left\{2+\sqrt{2},\ 2,\ 2-\sqrt{2},\ 0,\ 0\right\}$

In[1402]:= **Eigenvalues[A8]**

Out[1402]= $\left\{-\sqrt{2},\ \sqrt{2},\ -1,\ 1,\ 0\right\}$

In[1403]:= **VertexDegree[G8]**

Out[1403]= {2, 1, 1, 1, 1}

In[1404]:= **D8 = DiagonalMatrix[%]**

Out[1404]= {{2, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[1405]:= **MatrixForm[D8]**

Out[1405]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1406]:= **L8 = D8 - A8**

Out[1406]= {{2, -1, -1, 0, 0}, {-1, 1, 0, 0, 0},
{-1, 0, 1, 0, 0}, {0, 0, 0, 1, -1}, {0, 0, 0, -1, 1}}

In[1407]:= **MatrixForm[L8]**

Out[1407]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

In[1408]:= **Eigenvalues[L8]**

Out[1408]= {3, 2, 1, 0, 0}

In[1409]:= **Eigenvalues[A9]**

Out[1409]= {-2, 2, 0, 0, 0}

In[1410]:= **VertexDegree[G9]**

Out[1410]= {4, 1, 1, 1, 1}

In[1411]:= **D9 = DiagonalMatrix[%]**

Out[1411]= {{4, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[1412]:= **MatrixForm[D9]**

Out[1412]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1413]:= **L9 = D9 - A9**

Out[1413]= {{4, -1, -1, -1, -1}, {-1, 1, 0, 0, 0},
{-1, 0, 1, 0, 0}, {-1, 0, 0, 1, 0}, {-1, 0, 0, 0, 1}}

In[1414]:= **MatrixForm[L9]**

Out[1414]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1415]:= **Eigenvalues[L9]**

Out[1415]= {5, 1, 1, 1, 0}

In[1416]:= **N[Eigenvalues[A10]]**

Out[1416]= {2.17009, -1.48119, -1., 0.311108, 0.}

In[1417]:= **VertexDegree[G10]**

Out[1417]= {3, 2, 2, 1, 0}

In[1418]:= **D10 = DiagonalMatrix[%]**

Out[1418]= {{3, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1419]:= **MatrixForm[D10]**

Out[1419]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1420]:= **L10 = D10 - A10**

Out[1420]= {{3, -1, -1, -1, 0}, {-1, 2, -1, 0, 0},
{-1, -1, 2, 0, 0}, {-1, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}

In[1421]:= **MatrixForm[L10]**

Out[1421]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1422]:= **Eigenvalues[L10]**

Out[1422]= {4, 3, 1, 0, 0}

In[1423]:= **Eigenvalues[A11]**

Out[1423]= $\left\{-\sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2}}, -\sqrt{2-\sqrt{2}}, \sqrt{2-\sqrt{2}}, 0\right\}$

In[1424]:= **VertexDegree[G11]**

Out[1424]= {3, 2, 1, 1, 1}

In[1425]:= **D11 = DiagonalMatrix[%]**

Out[1425]= {{3, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[1426]:= **MatrixForm[D11]**

Out[1426]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1427]:= **L11 = D11 - A11**

Out[1427]= {{3, -1, -1, -1, 0}, {-1, 2, 0, 0, -1},
{-1, 0, 1, 0, 0}, {-1, 0, 0, 1, 0}, {0, -1, 0, 0, 1}}

In[1428]:= **MatrixForm[L11]**

Out[1428]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

In[1429]:= **Eigenvalues[L11]**

Out[1429]= $\{\text{Root}[-5 + 13\,\#1 - 7\,\#1^2 + \#1^3\,\&, 3],$
$\text{Root}[-5 + 13\,\#1 - 7\,\#1^2 + \#1^3\,\&, 2], 1, \text{Root}[-5 + 13\,\#1 - 7\,\#1^2 + \#1^3\,\&, 1], 0\}$

In[1430]:= **Eigenvalues[A12]**

Out[1430]= {2, -1, -1, -1, 1}

In[1431]:= **VertexDegree[G12]**

Out[1431]= {2, 2, 2, 1, 1}

In[1432]:= **D12 = DiagonalMatrix[%]**

Out[1432]= {{2, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[242]:= **MatrixForm[D12]**

Out[242]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1433]:= **L12 = D12 - A12**

Out[1433]= {{2, -1, -1, 0, 0}, {-1, 2, -1, 0, 0},
{-1, -1, 2, 0, 0}, {0, 0, 0, 1, -1}, {0, 0, 0, -1, 1}}

In[1434]:= **MatrixForm[L12]**

Out[1434]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

In[1435]:= **Eigenvalues[L12]**

Out[1435]= {3, 3, 2, 0, 0}

In[1436]:= **Eigenvalues[A13]**

Out[1436]= $\left\{-\sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2}}, -\sqrt{2-\sqrt{2}}, \sqrt{2-\sqrt{2}}, 0\right\}$

In[1437]:= **VertexDegree[G13]**

Out[1437]= {2, 3, 1, 1, 1}

In[1438]:= **D13 = DiagonalMatrix[%]**

Out[1438]= {{2, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[1439]:= **MatrixForm[D13]**

Out[1439]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1440]:= **L13 = D13 - A13**

Out[1440]= {{2, -1, -1, 0, 0}, {-1, 3, 0, -1, -1},
{-1, 0, 1, 0, 0}, {0, -1, 0, 1, 0}, {0, -1, 0, 0, 1}}

In[1441]:= **MatrixForm[L13]**

Out[1441]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

In[1442]:= **Eigenvalues[L13]**

Out[1442]= $\{\text{Root}[-5 + 13 \#1 - 7 \#1^2 + \#1^3 \&, 3], \text{Root}[-5 + 13 \#1 - 7 \#1^2 + \#1^3 \&, 2], 1, \text{Root}[-5 + 13 \#1 - 7 \#1^2 + \#1^3 \&, 1], 0\}$

In[1443]:= **Eigenvalues[A14]**

Out[1443]= {-2, 2, 0, 0, 0}

In[1444]:= **VertexDegree[G14]**

Out[1444]= {2, 2, 2, 2, 0}

In[1445]:= **D14 = DiagonalMatrix[%]**

Out[1445]= {{2, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 0}}

In[1446]:= **MatrixForm[D14]**

Out[1446]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1447]:= **L14 = D14 - A14**

Out[1447]= {{2, -1, -1, 0, 0}, {-1, 2, 0, -1, 0}, {-1, 0, 2, -1, 0}, {0, -1, -1, 2, 0}, {0, 0, 0, 0, 0}}

In[1448]:= **MatrixForm[L14]**

Out[1448]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1449]:= **Eigenvalues[L14]**

Out[1449]= {4, 2, 2, 0, 0}

In[1450]:= **N[Eigenvalues[A15]]**

Out[1450]= {2.34292, -1.81361, -1., 0.470683, 0.}

In[1451]:= **VertexDegree[G15]**

Out[1451]= {4, 2, 2, 1, 1}

In[1452]:= **D15 = DiagonalMatrix[%]**

Out[1452]= {{4, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[1453]:= **MatrixForm[D15]**

Out[1453]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1454]:= **L15 = D15 - A15**

Out[1454]= {{4, -1, -1, -1, -1}, {-1, 2, -1, 0, 0},
{-1, -1, 2, 0, 0}, {-1, 0, 0, 1, 0}, {-1, 0, 0, 0, 1}}

In[1455]:= **MatrixForm[L15]**

Out[1455]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1456]:= **Eigenvalues[L15]**

Out[1456]= {5, 3, 1, 1, 0}

In[1457]:= **Eigenvalues[A16]**

Out[1457]= $\left\{\frac{1}{2}\left(1+\sqrt{17}\right), \frac{1}{2}\left(1-\sqrt{17}\right), -1, 0, 0\right\}$

In[1458]:= **VertexDegree[G16]**

Out[1458]= {3, 3, 2, 2, 0}

In[1459]:= **D16 = DiagonalMatrix[%]**

Out[1459]= {{3, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 0}}

In[1460]:= **MatrixForm[D16]**

Out[1460]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1461]:= **L16 = D16 - A16**

Out[1461]= {{3, -1, -1, -1, 0}, {-1, 3, -1, -1, 0},
{-1, -1, 2, 0, 0}, {-1, -1, 0, 2, 0}, {0, 0, 0, 0, 0}}

In[1462]:= **MatrixForm[L16]**

Out[1462]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1463]:= **Eigenvalues[L16]**

Out[1463]= {4, 4, 2, 0, 0}

In[1464]:= **Eigenvalues[A17]**

Out[1464]= $\left\{\frac{1}{2}\left(1+\sqrt{13}\right), \frac{1}{2}\left(-1-\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{13}\right), \frac{1}{2}\left(-1+\sqrt{5}\right), 0\right\}$

In[1465]:= **VertexDegree[G17]**

Out[1465]= {3, 3, 2, 1, 1}

In[1466]:= **D17 = DiagonalMatrix[%]**

Out[1466]= {{3, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}}

In[1467]:= **MatrixForm[D17]**

Out[1467]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1468]:= **L17 = D17 - A17**

Out[1468]= {{3, -1, -1, -1, 0}, {-1, 3, -1, 0, -1},
{-1, -1, 2, 0, 0}, {-1, 0, 0, 1, 0}, {0, -1, 0, 0, 1}}

In[1469]:= **MatrixForm[L17]**

Out[1469]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

In[1470]:= **Eigenvalues[L17]**

Out[1470]= $\left\{\frac{1}{2}\left(5+\sqrt{13}\right), \frac{1}{2}\left(5+\sqrt{5}\right), \frac{1}{2}\left(5-\sqrt{5}\right), \frac{1}{2}\left(5-\sqrt{13}\right), 0\right\}$

In[1471]:= **N[Eigenvalues[A18]]**

Out[1471]= {2.21432, -1.67513, -1., 1., -0.539189}

In[1472]:= **VertexDegree[G18]**

Out[1472]= {3, 2, 2, 2, 1}

In[1473]:= **D18 = DiagonalMatrix[%]**

Out[1473]= {{3, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 1}}

In[1474]:= **MatrixForm[D18]**

Out[1474]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1475]:= **L18 = D18 - A18**

Out[1475]= {{3, -1, -1, -1, 0}, {-1, 2, -1, 0, 0},
{-1, -1, 2, 0, 0}, {-1, 0, 0, 2, -1}, {0, 0, 0, -1, 1}}

In[1476]:= **MatrixForm[L18]**

Out[1476]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

In[1477]:= **Eigenvalues[L18]**

Out[1477]= $\{\text{Root}[-5+13\,\sharp1-7\,\sharp1^2+\sharp1^3\,\&,3],3,\text{Root}[-5+13\,\sharp1-7\,\sharp1^2+\sharp1^3\,\&,2],\text{Root}[-5+13\,\sharp1-7\,\sharp1^2+\sharp1^3\,\&,1],0\}$

In[1478]:= **Eigenvalues[A19]**

Out[1478]= $\left\{-\sqrt{\frac{1}{2}\left(5+\sqrt{17}\right)},\sqrt{\frac{1}{2}\left(5+\sqrt{17}\right)},-\sqrt{\frac{1}{2}\left(5-\sqrt{17}\right)},\sqrt{\frac{1}{2}\left(5-\sqrt{17}\right)},0\right\}$

In[1479]:= **VertexDegree[G19]**

Out[1479]= {3, 2, 2, 1, 2}

In[1480]:= **D19 = DiagonalMatrix[%]**

Out[1480]= {{3, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 2}}

In[1481]:= **MatrixForm[D19]**

Out[1481]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[1482]:= **L19 = D19 - A19**

Out[1482]= {{3, -1, -1, -1, 0}, {-1, 2, 0, 0, -1}, {-1, 0, 2, 0, -1}, {-1, 0, 0, 1, 0}, {0, -1, -1, 0, 2}}

In[1483]:= **MatrixForm[L19]**

Out[1483]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 2 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 2 \end{pmatrix}$$

In[1484]:= **Eigenvalues[L19]**

Out[1484]= $\{\text{Root}[-10+18\,\sharp1-8\,\sharp1^2+\sharp1^3\,\&,3],\text{Root}[-10+18\,\sharp1-8\,\sharp1^2+\sharp1^3\,\&,2],2,\text{Root}[-10+18\,\sharp1-8\,\sharp1^2+\sharp1^3\,\&,1],0\}$

In[1485]:= **Eigenvalues[A20]**

Out[1485]= $\left\{2,\frac{1}{2}\left(-1-\sqrt{5}\right),\frac{1}{2}\left(-1-\sqrt{5}\right),\frac{1}{2}\left(-1+\sqrt{5}\right),\frac{1}{2}\left(-1+\sqrt{5}\right)\right\}$

In[1486]:= **VertexDegree[G20]**

Out[1486]= {2, 2, 2, 2, 2}

In[1487]:= **D20 = DiagonalMatrix[%]**

Out[1487]= {{2, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 2}}

In[1488]:=
```
MatrixForm[D20]
```

Out[1488]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[1489]:=
```
L20 = D20 - A20
```

Out[1489]=
```
{{2, -1, -1, 0, 0}, {-1, 2, 0, -1, 0},
 {-1, 0, 2, 0, -1}, {0, -1, 0, 2, -1}, {0, 0, -1, -1, 2}}
```

In[1490]:=
```
MatrixForm[L20]
```

Out[1490]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{pmatrix}$$

In[1491]:=
```
Eigenvalues[L20]
```

Out[1491]= $\left\{\frac{1}{2}\left(5+\sqrt{5}\right), \frac{1}{2}\left(5+\sqrt{5}\right), \frac{1}{2}\left(5-\sqrt{5}\right), \frac{1}{2}\left(5-\sqrt{5}\right), 0\right\}$

In[1492]:=
```
N[Eigenvalues[A21]]
```

Out[1492]=
```
{2.68554, -1.74912, -1.27133, 0.334904, 0.}
```

In[1493]:=
```
VertexDegree[G21]
```

Out[1493]=
```
{4, 3, 2, 2, 1}
```

In[1494]:=
```
D21 = DiagonalMatrix[%]
```

Out[1494]=
```
{{4, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 1}}
```

In[1495]:=
```
MatrixForm[D21]
```

Out[1495]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1496]:=
```
L21 = D21 - A21
```

Out[1496]=
```
{{4, -1, -1, -1, -1}, {-1, 3, -1, -1, 0},
 {-1, -1, 2, 0, 0}, {-1, -1, 0, 2, 0}, {-1, 0, 0, 0, 1}}
```

In[1497]:=
```
MatrixForm[L21]
```

Out[1497]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & -1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1498]:=
```
Eigenvalues[L21]
```

Out[1498]=
```
{5, 4, 2, 1, 0}
```

In[1499]:= **Eigenvalues[A22]**

Out[1499]= $\left\{\frac{1}{2}\left(1+\sqrt{17}\right), \frac{1}{2}\left(1-\sqrt{17}\right), -1, -1, 1\right\}$

In[1500]:= **VertexDegree[G22]**

Out[1500]= {4, 2, 2, 2, 2}

In[1501]:= **D22 = DiagonalMatrix[%]**

Out[1501]= {{4, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 2}}

In[1502]:= **MatrixForm[D22]**

Out[1502]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[1503]:= **L22 = D22 - A22**

Out[1503]= {{4, -1, -1, -1, -1}, {-1, 2, -1, 0, 0},
{-1, -1, 2, 0, 0}, {-1, 0, 0, 2, -1}, {-1, 0, 0, -1, 2}}

In[1504]:= **MatrixForm[L22]**

Out[1504]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}$$

In[1505]:= **Eigenvalues[L22]**

Out[1505]= {5, 3, 3, 1, 0}

In[1506]:= **Eigenvalues[A23]**

Out[1506]= {3, -1, -1, -1, 0}

In[1507]:= **VertexDegree[G23]**

Out[1507]= {3, 3, 3, 3, 0}

In[1508]:= **D23 = DiagonalMatrix[%]**

Out[1508]= {{3, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 3, 0, 0}, {0, 0, 0, 3, 0}, {0, 0, 0, 0, 0}}

In[1509]:= **MatrixForm[D23]**

Out[1509]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1510]:= **L23 = D23 - A23**

Out[1510]= {{3, -1, -1, -1, 0}, {-1, 3, -1, -1, 0},
{-1, -1, 3, -1, 0}, {-1, -1, -1, 3, 0}, {0, 0, 0, 0, 0}}

In[1511]:= **MatrixForm[L23]**

Out[1511]//MatrixForm=
$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1512]:= **Eigenvalues[L23]**

Out[1512]= {4, 4, 4, 0, 0}

In[1513]:= **N[Eigenvalues[A24]]**

Out[1513]= {2.64119, -1.77571, -1., 0.723742, -0.589216}

In[1514]:= **VertexDegree[G24]**

Out[1514]= {3, 3, 3, 2, 1}

In[1515]:= **D24 = DiagonalMatrix[%]**

Out[1515]= {{3, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 3, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 1}}

In[1516]:= **MatrixForm[D24]**

Out[1516]//MatrixForm=
$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1517]:= **L24 = D24 - A24**

Out[1517]= {{3, -1, -1, -1, 0}, {-1, 3, -1, -1, 0},
{-1, -1, 3, 0, -1}, {-1, -1, 0, 2, 0}, {0, 0, -1, 0, 1}}

In[1518]:= **MatrixForm[L24]**

Out[1518]//MatrixForm=
$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & 0 & -1 \\ -1 & -1 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

In[1519]:= **Eigenvalues[L24]**

Out[1519]= {Root[-10 + 18 ⌗1 - 8 ⌗1^2 + ⌗1^3 &, 3], 4,
Root[-10 + 18 ⌗1 - 8 ⌗1^2 + ⌗1^3 &, 2], Root[-10 + 18 ⌗1 - 8 ⌗1^2 + ⌗1^3 &, 1], 0}

In[1520]:= **N[Eigenvalues[A25]]**

Out[1520]= {2.48119, -2., -1.17009, 0.688892, 0.}

In[1521]:= **VertexDegree[G25]**

Out[1521]= {3, 3, 2, 2, 2}

In[1522]:= **D25 = DiagonalMatrix[%]**

Out[1522]= {{3, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 2}}

In[1523]:= **MatrixForm[D25]**

Out[1523]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[1524]:= **L25 = D25 - A25**

Out[1524]= {{3, -1, -1, -1, 0}, {-1, 3, -1, 0, -1},
{-1, -1, 2, 0, 0}, {-1, 0, 0, 2, -1}, {0, -1, 0, -1, 2}}

In[1525]:= **MatrixForm[L25]**

Out[1525]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{pmatrix}$$

In[1526]:= **Eigenvalues[L25]**

Out[1526]= $\left\{\frac{1}{2}\left(7+\sqrt{5}\right), \frac{1}{2}\left(5+\sqrt{5}\right), \frac{1}{2}\left(7-\sqrt{5}\right), \frac{1}{2}\left(5-\sqrt{5}\right), 0\right\}$

In[1527]:= **Eigenvalues[A26]**

Out[1527]= $\left\{-\sqrt{6}, \sqrt{6}, 0, 0, 0\right\}$

In[1528]:= **VertexDegree[G26]**

Out[1528]= {3, 2, 2, 2, 3}

In[1529]:= **D26 = DiagonalMatrix[%]**

Out[1529]= {{3, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 3}}

In[1531]:= **MatrixForm[D26]**

Out[1531]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

In[1532]:= **L26 = D26 - A26**

Out[1532]= {{3, -1, -1, -1, 0}, {-1, 2, 0, 0, -1},
{-1, 0, 2, 0, -1}, {-1, 0, 0, 2, -1}, {0, -1, -1, -1, 3}}

In[1533]:= **MatrixForm[L26]**

Out[1533]//MatrixForm=

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 2 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{pmatrix}$$

In[1534]:= `Eigenvalues[L26]`

Out[1534]= {5, 3, 2, 2, 0}

In[1535]:= `Eigenvalues[A27]`

Out[1535]= {3, -2, -1, 0, 0}

In[1536]:= `VertexDegree[G27]`

Out[1536]= {4, 4, 2, 2, 2}

In[1537]:= `D27 = DiagonalMatrix[%]`

Out[1537]= {{4, 0, 0, 0, 0}, {0, 4, 0, 0, 0}, {0, 0, 2, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 2}}

In[1538]:= `MatrixForm[D27]`

Out[1538]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[1539]:= `L27 = D27 - A27`

Out[1539]= {{4, -1, -1, -1, -1}, {-1, 4, -1, -1, -1},
{-1, -1, 2, 0, 0}, {-1, -1, 0, 2, 0}, {-1, -1, 0, 0, 2}}

In[1540]:= `MatrixForm[L27]`

Out[1540]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & -1 & 0 & 2 & 0 \\ -1 & -1 & 0 & 0 & 2 \end{pmatrix}$$

In[1541]:= `Eigenvalues[L27]`

Out[1541]= {5, 5, 2, 2, 0}

In[1542]:= `N[Eigenvalues[A28]]`

Out[1542]= {3.08613, -1.51414, -1., -1., 0.428007}

In[1543]:= `VertexDegree[G28]`

Out[1543]= {4, 3, 3, 3, 1}

In[1544]:= `D28 = DiagonalMatrix[%]`

Out[1544]= {{4, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 3, 0, 0}, {0, 0, 0, 3, 0}, {0, 0, 0, 0, 1}}

In[1545]:= `MatrixForm[D28]`

Out[1545]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1546]:= **L28 = D28 - A28**

Out[1546]= {{4, -1, -1, -1, -1}, {-1, 3, -1, -1, 0}, {-1, -1, 3, -1, 0}, {-1, -1, -1, 3, 0}, {-1, 0, 0, 0, 1}}

In[1547]:= **MatrixForm[L28]**

Out[1547]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1548]:= **Eigenvalues[L28]**

Out[1548]= {5, 4, 4, 1, 0}

In[1549]:= **N[Eigenvalues[A29]]**

Out[1549]= {2.93543, -1.61803, -1.47283, 0.618034, -0.462598}

In[1550]:= **VertexDegree[G29]**

Out[1550]= {4, 3, 3, 2, 2}

In[1551]:= **D29 = DiagonalMatrix[%]**

Out[1551]= {{4, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 3, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 2}}

In[1552]:= **MatrixForm[D29]**

Out[1552]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[1553]:= **L29 = D29 - A29**

Out[1553]= {{4, -1, -1, -1, -1}, {-1, 3, -1, -1, 0}, {-1, -1, 3, 0, -1}, {-1, -1, 0, 2, 0}, {-1, 0, -1, 0, 2}}

In[1554]:= **MatrixForm[L29]**

Out[1554]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & 0 & -1 \\ -1 & -1 & 0 & 2 & 0 \\ -1 & 0 & -1 & 0 & 2 \end{pmatrix}$$

In[1555]:= **Eigenvalues[L29]**

Out[1555]= $\left\{5,\ 3+\sqrt{2},\ 3,\ 3-\sqrt{2},\ 0\right\}$

In[1556]:= **N[Eigenvalues[A30]]**

Out[1556]= {2.85577, -2.17741, -1., 0.321637, 0.}

In[1557]:= **VertexDegree[G30]**

Out[1557]= {3, 3, 3, 3, 2}

```
In[1558]:= D30 = DiagonalMatrix[%]

Out[1558]= {{3, 0, 0, 0, 0}, {0, 3, 0, 0, 0}, {0, 0, 3, 0, 0}, {0, 0, 0, 3, 0}, {0, 0, 0, 0, 2}}

In[1559]:= MatrixForm[D30]

Out[1559]//MatrixForm=
```

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

```
In[1560]:= L30 = D30 - A30

Out[1560]= {{3, -1, -1, -1, 0}, {-1, 3, -1, -1, 0},
            {-1, -1, 3, 0, -1}, {-1, -1, 0, 3, -1}, {0, 0, -1, -1, 2}}

In[1561]:= MatrixForm[L30]

Out[1561]//MatrixForm=
```

$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & 0 & -1 \\ -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{pmatrix}$$

```
In[1562]:= Eigenvalues[L30]

Out[1562]= {5, 4, 3, 2, 0}

In[244]:= N[Eigenvalues[A31]]

Out[244]= {3.3234, -1.68133, -1., -1., 0.357926}

In[354]:= VertexDegree[G31]

Out[354]= {4, 4, 3, 3, 2}

In[1563]:= D31 = DiagonalMatrix[%]

Out[1563]= {{5, 0, 0, 0, 0}, {0, 4, 0, 0, 0}, {0, 0, 3, 0, 0}, {0, 0, 0, 2, 0}, {0, 0, 0, 0, 0}}

In[1564]:= MatrixForm[D31]

Out[1564]//MatrixForm=
```

$$\begin{pmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[1565]:= L31 = D31 - A31

Out[1565]= {{5, -1, -1, -1, -1}, {-1, 4, -1, -1, -1},
            {-1, -1, 3, -1, 0}, {-1, -1, -1, 2, 0}, {-1, -1, 0, 0, 0}}

In[1566]:= MatrixForm[L31]

Out[1566]//MatrixForm=
```

$$\begin{pmatrix} 5 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ -1 & -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

In[1568]:= **Eigenvalues[L31]**

Out[1568]= $\{\text{Root}[55-47\#1-83\#1^2+63\#1^3-14\#1^4+\#1^5\ \&,5],$
$\text{Root}[55-47\#1-83\#1^2+63\#1^3-14\#1^4+\#1^5\ \&,4],$
$\text{Root}[55-47\#1-83\#1^2+63\#1^3-14\#1^4+\#1^5\ \&,3],$
$\text{Root}[55-47\#1-83\#1^2+63\#1^3-14\#1^4+\#1^5\ \&,1],$
$\text{Root}[55-47\#1-83\#1^2+63\#1^3-14\#1^4+\#1^5\ \&,2]\}$

In[1569]:= **Eigenvalues[A32]**

Out[1569]= $\{1+\sqrt{5},-2,1-\sqrt{5},0,0\}$

In[360]:= **VertexDegree[G32]**

Out[360]= {4, 3, 3, 3, 3}

In[1570]:= **D32 = DiagonalMatrix[%]**

Out[1570]= $\{\{1+\sqrt{5},0,0,0,0\},\{0,-2,0,0,0\},$
$\{0,0,1-\sqrt{5},0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\}\}$

In[1571]:= **MatrixForm[D32]**

Out[1571]//MatrixForm=
$$\begin{pmatrix} 1+\sqrt{5} & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1-\sqrt{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1572]:= **L32 = D32 - A32**

Out[1572]= $\{\{1+\sqrt{5},-1,-1,-1,-1\},\{-1,-2,-1,-1,0\},$
$\{-1,-1,1-\sqrt{5},0,-1\},\{-1,-1,0,0,-1\},\{-1,0,-1,-1,0\}\}$

In[1573]:= **MatrixForm[L32]**

Out[1573]//MatrixForm=
$$\begin{pmatrix} 1+\sqrt{5} & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & -1 & 0 \\ -1 & -1 & 1-\sqrt{5} & 0 & -1 \\ -1 & -1 & 0 & 0 & -1 \\ -1 & 0 & -1 & -1 & 0 \end{pmatrix}$$

In[1574]:= **Eigenvalues[L32]**

Out[1574]= $\{\text{Root}[\{-5+\#1^2\ \&,4+4\#1+22\#2+2\#1\#2-\#2^2-\#1\#2^2-16\#2^3+\#2^5\ \&\},\{2,5\}],$
$\text{Root}[\{-5+\#1^2\ \&,4+4\#1+22\#2+2\#1\#2-\#2^2-\#1\#2^2-16\#2^3+\#2^5\ \&\},\{2,1\}],$
$\text{Root}[\{-5+\#1^2\ \&,4+4\#1+22\#2+2\#1\#2-\#2^2-\#1\#2^2-16\#2^3+\#2^5\ \&\},\{2,4\}],$
$\text{Root}[\{-5+\#1^2\ \&,4+4\#1+22\#2+2\#1\#2-\#2^2-\#1\#2^2-16\#2^3+\#2^5\ \&\},\{2,2\}],$
$\text{Root}[\{-5+\#1^2\ \&,4+4\#1+22\#2+2\#1\#2-\#2^2-\#1\#2^2-16\#2^3+\#2^5\ \&\},\{2,3\}]\}$

In[1575]:= **Eigenvalues[A33]**

Out[1575]= $\{1+\sqrt{7},1-\sqrt{7},-1,-1,0\}$

```
In[1576]:= VertexDegree[G33]

Out[1576]= {4, 4, 4, 3, 3}

In[1577]:= D33 = DiagonalMatrix[%]

Out[1577]= {{4, 0, 0, 0, 0}, {0, 4, 0, 0, 0}, {0, 0, 4, 0, 0}, {0, 0, 0, 3, 0}, {0, 0, 0, 0, 3}}

In[1578]:= MatrixForm[D33]

Out[1578]//MatrixForm=
```

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

```
In[1579]:= L33 = D33 - A33

Out[1579]= {{4, -1, -1, -1, -1}, {-1, 4, -1, -1, -1},
  {-1, -1, 4, -1, -1}, {-1, -1, -1, 3, 0}, {-1, -1, -1, 0, 3}}

In[1580]:= MatrixForm[L33]

Out[1580]//MatrixForm=
```

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 3 & 0 \\ -1 & -1 & -1 & 0 & 3 \end{pmatrix}$$

```
In[1581]:= Eigenvalues[L33]

Out[1581]= {5, 5, 5, 3, 0}

In[1582]:= Eigenvalues[A34]

Out[1582]= {4, -1, -1, -1, -1}

In[1583]:= VertexDegree[G34]

Out[1583]= {4, 4, 4, 4, 4}

In[1584]:= D34 = DiagonalMatrix[%]

Out[1584]= {{4, 0, 0, 0, 0}, {0, 4, 0, 0, 0}, {0, 0, 4, 0, 0}, {0, 0, 0, 4, 0}, {0, 0, 0, 0, 4}}

In[1585]:= MatrixForm[D34]

Out[1585]//MatrixForm=
```

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

```
In[1586]:= L34 = D34 - A34

Out[1586]= {{4, -1, -1, -1, -1}, {-1, 4, -1, -1, -1},
  {-1, -1, 4, -1, -1}, {-1, -1, -1, 4, -1}, {-1, -1, -1, -1, 4}}
```

In[1587]:=
```
MatrixForm[L34]
```

Out[1587]//MatrixForm=

$$\begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{pmatrix}$$

In[1588]:=
```
Eigenvalues[L34]
```

Out[1588]= {5, 5, 5, 5, 0}

#Determinare i grafi su 5 vertici connessi, indicando il loro numero nella numerazione riportata nel file Lezione10.pdf.

Verifichiamo ora quali sono i grafi connessi e ne indichiamoo il numero nella numerazione riportata nel file Lezione10.pdf

In[1]:=
```
M = {}
For[i = 0, i < 1024, i++,
 A = Insert[IntegerDigits[i, 2, 10], 0, {{1}, {5}, {5}, {8},
    {8}, {8}, {10}, {10}, {10}, {10}, {11}, {11}, {11}, {11}, {11}}];
 B = Partition[A, 5];
 BB = Transpose[B] + B;
 M = Append[M, BB]
]
```

Out[1]= { }

In[1600]:=
```
ConnectedGraphQ[G1]
```

Out[1600]= False

In[1599]:=
```
ConnectedGraphQ[G2]
```

Out[1599]= False

In[1598]:=
```
ConnectedGraphQ[G3]
```

Out[1598]= False

In[1597]:=
```
ConnectedGraphQ[G4]
```

Out[1597]= False

In[1596]:=
```
ConnectedGraphQ[G5]
```

Out[1596]= False

In[1594]:=
```
ConnectedGraphQ[G6]
```

Out[1594]= False

In[1593]:=
```
ConnectedGraphQ[G7]
```

Out[1593]= False

In[1592]:=
```
ConnectedGraphQ[G8]
```

Out[1592]= False

In[1591]:=
```
ConnectedGraphQ[G9]
```

Out[1591]= True

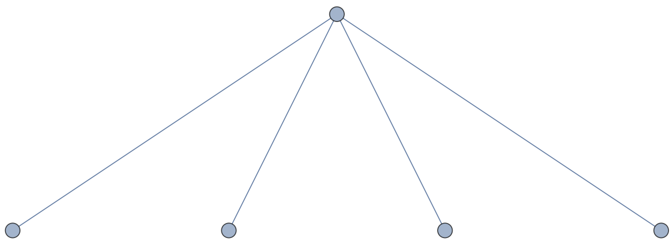
In[1604]:= **MatrixForm[M[[961]]]**

Out[1604]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1605]:= **AdjacencyGraph[M[[961]]]**

Out[1605]=



In[1608]:= **ConnectedGraphQ[G10]**

Out[1608]= False

In[1609]:= **ConnectedGraphQ[G11]**

Out[1609]= True

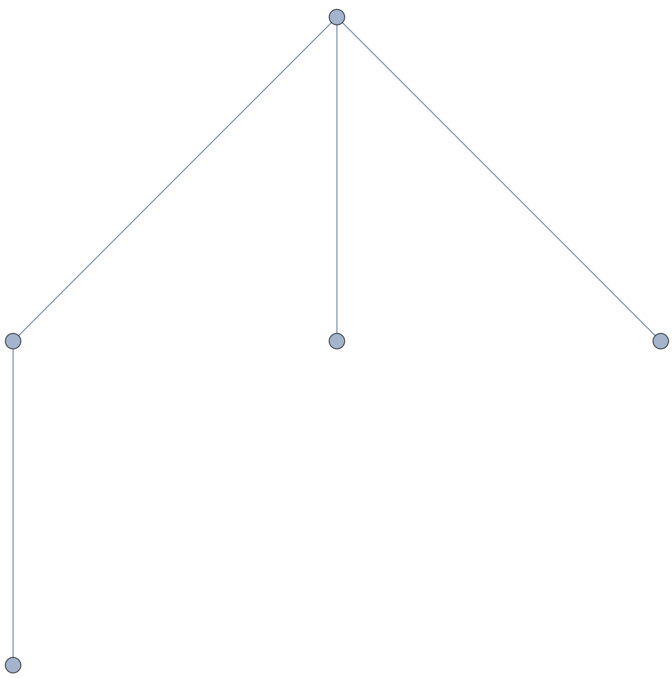
In[1616]:= **MatrixForm[M[[905]]]**

Out[1616]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1617]:= **AdjacencyGraph[M[[905]]]**

Out[1617]=



In[1618]:= `ConnectedGraphQ[G12]`

Out[1618]= False

In[1619]:= `ConnectedGraphQ[G13]`

Out[1619]= True

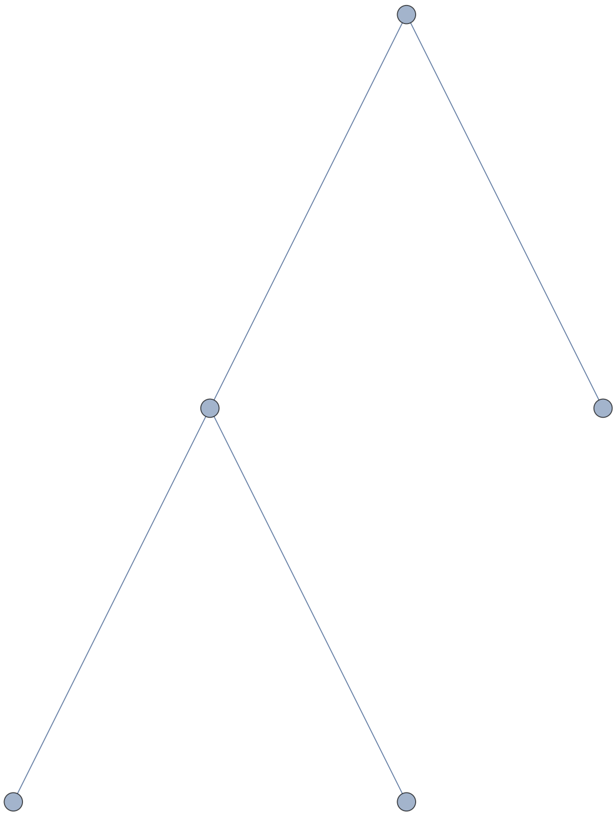
In[1623]:= `MatrixForm[M[[793]]]`

Out[1623]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1624]:= `AdjacencyGraph[M[[793]]]`

Out[1624]=



In[1626]:= `ConnectedGraphQ[G14]`

Out[1626]= False

In[1627]:= `ConnectedGraphQ[G15]`

Out[1627]= True

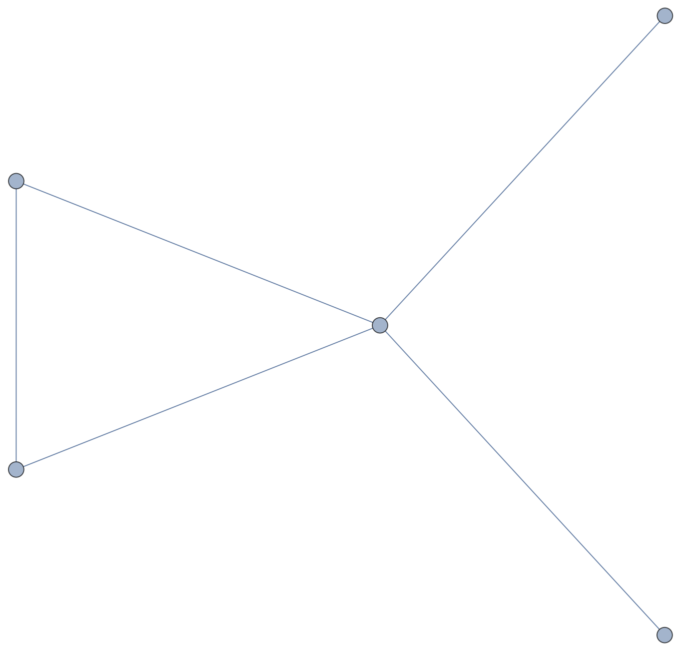
In[875]:= `MatrixForm[M[[993]]]`

Out[875]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[877]:= `AdjacencyGraph[M[[993]]]`

Out[877]=



In[1632]:= `ConnectedGraphQ[G16]`

Out[1632]= `False`

In[1634]:= `ConnectedGraphQ[G17]`

Out[1634]= `True`

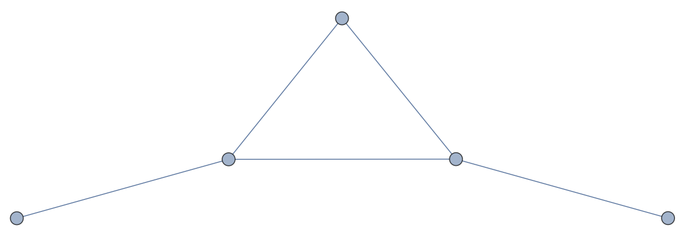
In[1644]:= `MatrixForm[M[[937]]]`

Out[1644]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1645]:= `AdjacencyGraph[M[[937]]]`

Out[1645]=



In[1646]:= `ConnectedGraphQ[G18]`

Out[1646]= `True`

In[1651]:= **MatrixForm[M[[930]]]**

Out[1651]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[1652]:= **AdjacencyGraph[M[[930]]]**

Out[1652]=

In[1653]:= **ConnectedGraphQ[G19]**

Out[1653]= True

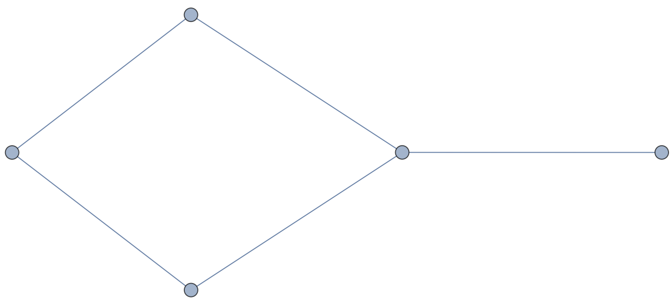
In[1657]:= **MatrixForm[M[[907]]]**

Out[1657]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

In[1658]:= **AdjacencyGraph[M[[907]]]**

Out[1658]=



In[1659]:= **ConnectedGraphQ[G20]**

Out[1659]= True

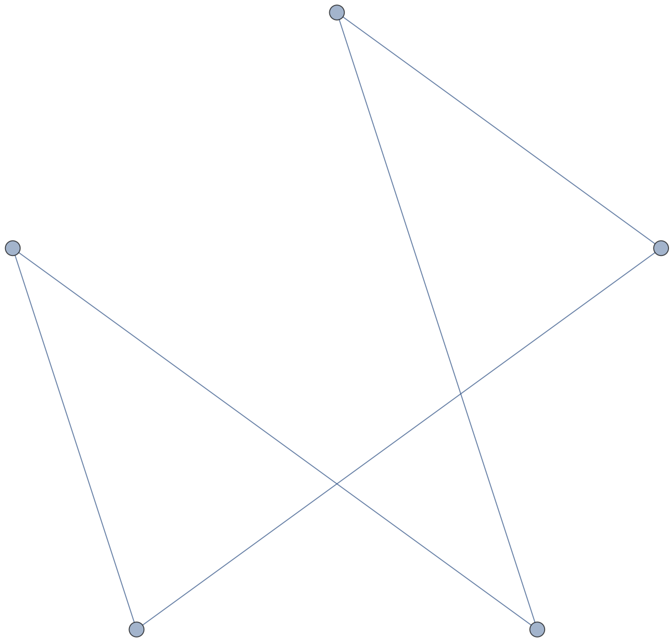
In[1666]:= **MatrixForm[M[[788]]]**

Out[1666]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

In[1665]:= **AdjacencyGraph[M[[788]]]**

Out[1665]=



In[1668]:= **ConnectedGraphQ[G21]**

Out[1668]= True

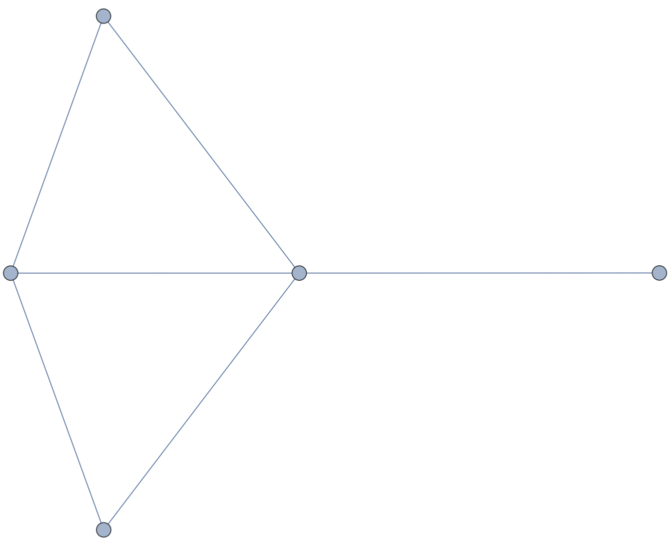
In[900]:= **MatrixForm[M[[1009]]]**

Out[900]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[901]:= **AdjacencyGraph[M[[1009]]]**

Out[901]=



In[1673]:= **ConnectedGraphQ[G22]**

Out[1673]= True

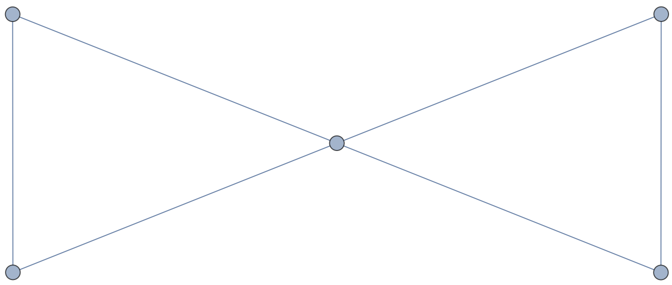
In[1679]:= `MatrixForm[M[[994]]]`

Out[1679]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[1681]:= `AdjacencyGraph[M[[994]]]`

Out[1681]=



In[1675]:= `ConnectedGraphQ[G23]`

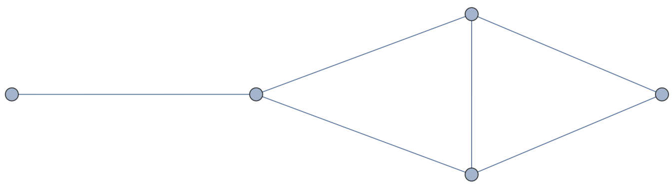
Out[1675]= `False`

In[1676]:= `ConnectedGraphQ[G24]`

Out[1676]= `True`

In[1686]:= `AdjacencyGraph[M[[947]]]`

Out[1686]=



In[1687]:= `ConnectedGraphQ[G25]`

Out[1687]= `True`

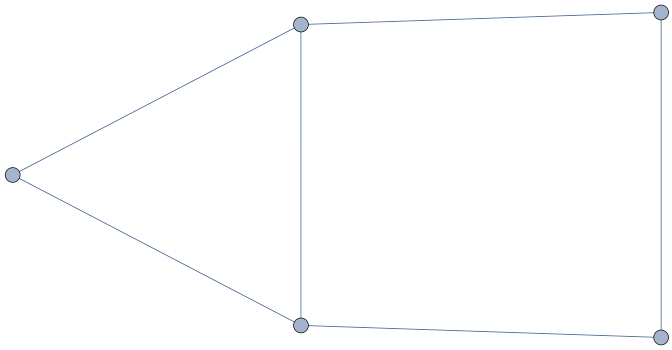
In[1692]:= `MatrixForm[M[[938]]]`

Out[1692]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

In[1693]:= `AdjacencyGraph[M[[938]]]`

Out[1693]=



In[1694]:= `ConnectedGraphQ[G26]`

Out[1694]= True

In[1698]:=

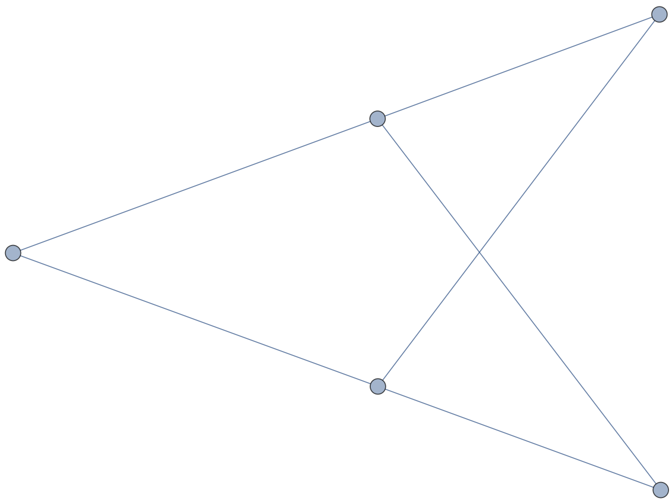
`MatrixForm[M[[908]]]`

Out[1698]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

In[1699]:= `AdjacencyGraph[M[[908]]]`

Out[1699]=



In[1701]:= `ConnectedGraphQ[G27]`

Out[1701]= True

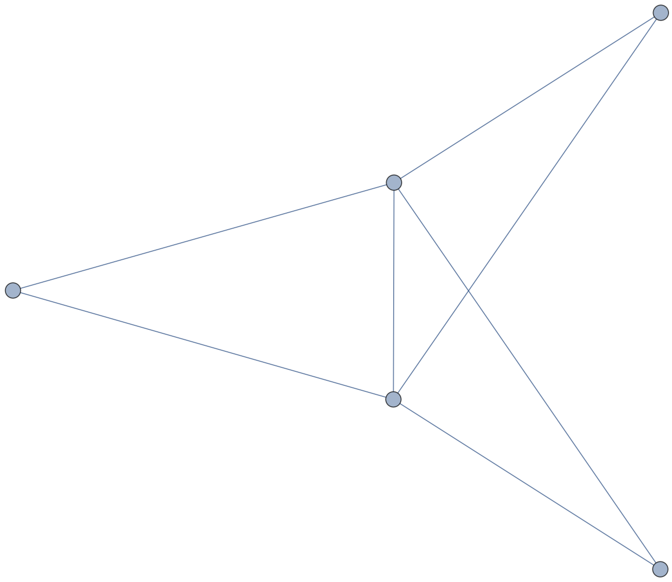
In[1706]:= `MatrixForm[M[[1017]]]`

Out[1706]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[1707]:= `AdjacencyGraph[M[[1017]]]`

Out[1707]=



In[1705]:= `ConnectedGraphQ[G28]`

Out[1705]= `True`

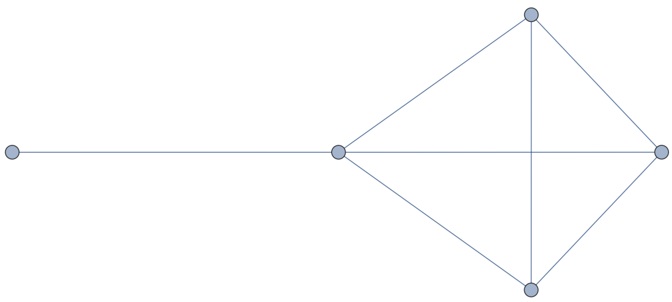
In[1711]:= `MatrixForm[M[[1013]]]`

Out[1711]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1714]:= `AdjacencyGraph[M[[1013]]]`

Out[1714]=



In[1715]:= `ConnectedGraphQ[G29]`

Out[1715]= `True`

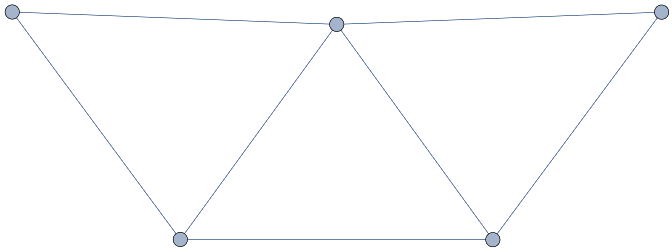
In[1719]:= `MatrixForm[M[[1011]]]`

Out[1719]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

In[1720]:= **AdjacencyGraph[M[[1011]]]**

Out[1720]=



In[1721]:= **ConnectedGraphQ[G30]**

Out[1721]= True

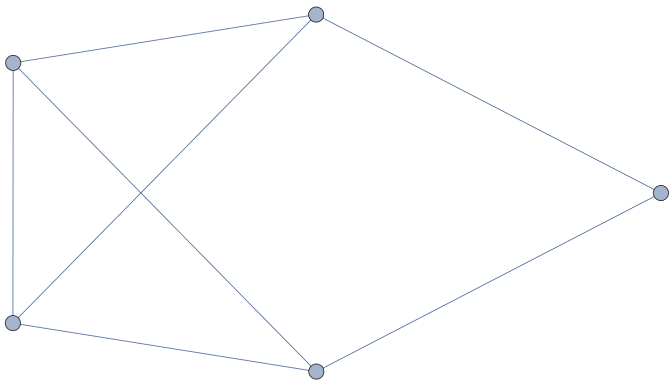
In[1725]:= **MatrixForm[M[[948]]]**

Out[1725]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

In[1726]:= **AdjacencyGraph[M[[948]]]**

Out[1726]=



In[1727]:= **ConnectedGraphQ[G31]**

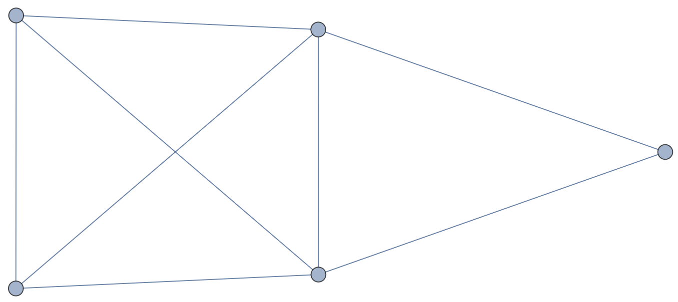
Out[1727]= True

In[967]:= **MatrixForm[M[[1021]]]**

Out[967]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

In[968]:=
```
AdjacencyGraph[M[[1021]]]
```

Out[968]=


In[1732]:=
```
ConnectedGraphQ[G32]
```

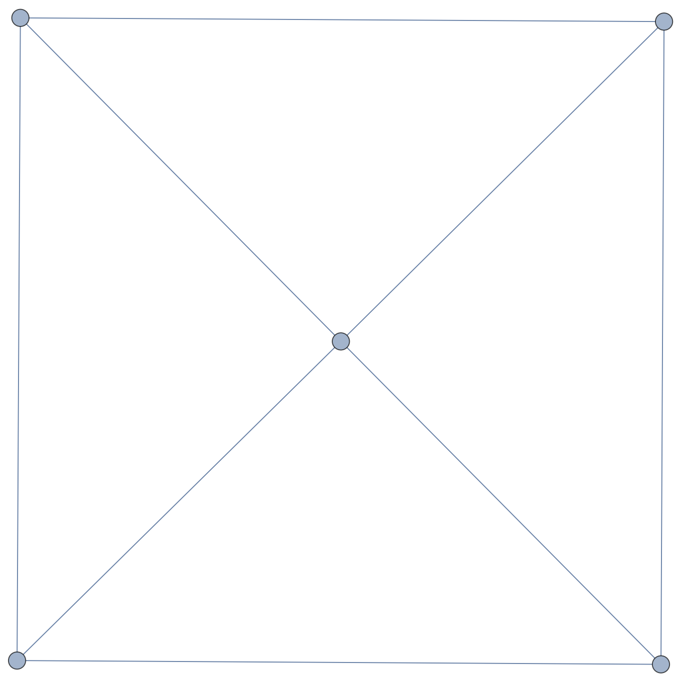
Out[1732]= True

In[972]:=
```
MatrixForm[M[[511]]]
```

Out[972]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

In[973]:=
```
AdjacencyGraph[M[[511]]]
```

Out[973]=


In[1734]:=
```
ConnectedGraphQ[G33]
```

Out[1734]= True

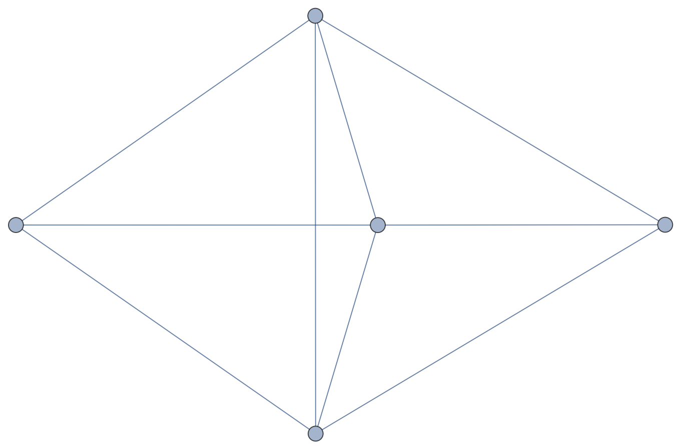
In[995]:= `MatrixForm[M[[512]]]`

Out[995]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

In[996]:= `AdjacencyGraph[M[[512]]]`

Out[996]=



In[1736]:= `ConnectedGraphQ[G34]`

Out[1736]= True

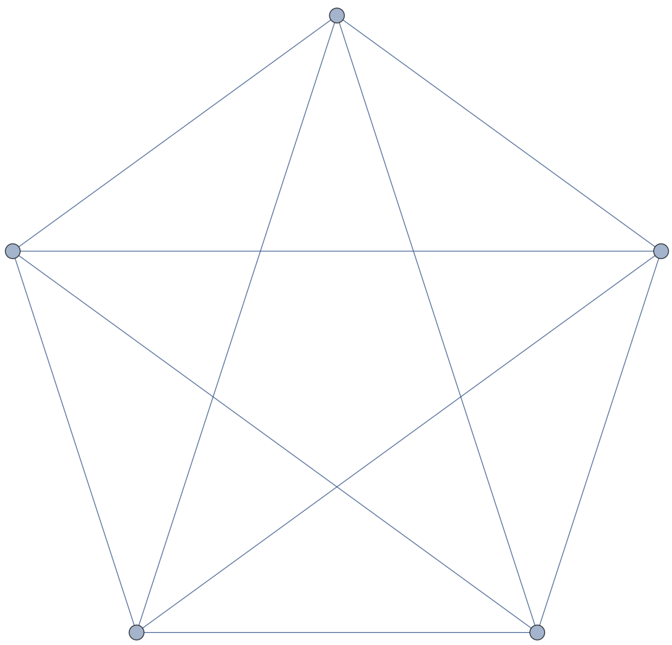
In[1737]:= `MatrixForm[M[[1024]]]`

Out[1737]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

In[1738]:=
```
AdjacencyGraph[M[[1024]]]
```

Out[1738]=



#Analogamente al punto precedente, determinare i grafi su 5 vertici che sono:
regolari (un grafo regolare è quello dove ogni vertice ha lo stesso numero di vicini, cioè ogni vertice ha lo stesso grado);
completi (un grafo completo è quello per cui ogni vertice è collegato a tutti i vertici rimanenti);
bipartiti (un grafo bipartito è tale che l'insieme dei suoi vertici si può partizionare in due sottoinsiemi tali che ogni vertice di una di queste due parti è collegato solo a vertici dell'altra);
lineari(un grafo tale che i suoi vertici possano essere scritti nell'ordine v1,v2,..vn tali che gli spigoli siano {vi, vi+1} con i = 1, 2, …, n − 1.), per determinarli utilizzeremo la funzione PathGraphQ
planari (un grafo planare può essere raffigurato in un piano in modo che non si abbiano archi che si intersecano);
circolari (un grafo circolare o grafo semplice consiste di un unico ciclo o, in altre parole, di un certo numero di vertici connessi in una catena chiusa);
alberi (un grafo non orientato nel quale due vertici qualsiasi sono connessi da uno e un solo cammino).

```
gap> A1:=[[0,0,0,0,0],[0,0,0,0,0],[0,0,0,0,0],[0,0,0,0,0],[0,0,0,0,0]];
[ [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G1:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A1[x][y]=1; end, true);
rec( adjacencies := [ [  ], [  ], [  ], [  ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G1);
true
gap> IsCompleteGraph(G1);
false
gap> IsBipartite(G1);
true
```

In[1740]:=
```
PathGraphQ[G1]
```

Out[1740]= False

In[1744]:= **PlanarGraphQ[G1]**

Out[1744]= True

In[1745]:= **SimpleGraphQ[G1]**

Out[1745]= True

In[1746]:= **TreeGraphQ[G1]**

Out[1746]= False

```
gap> A2:=[[0,1,0,0,0],[1,0,0,0,0],[0,0,0,0,0],[0,0,0,0,0],[0,0,0,0,0]];
[ [ 0, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G2:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A2[x][y]=1; end, true);
rec( adjacencies := [ [ 2 ], [ 1 ], [  ], [  ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G2);
false
gap> IsCompleteGraph(G2);
false
gap> IsBipartite(G2);
true
```

In[1747]:= **PathGraphQ[G2]**

Out[1747]= False

In[1748]:= **PlanarGraphQ[G2]**

Out[1748]= True

In[1749]:= **SimpleGraphQ[G2]**

Out[1749]= True

In[1750]:= **TreeGraphQ[G2]**

Out[1750]= False

```
gap> A3:=[[0,1,1,0,0],[1,0,0,0,0],[1,0,0,0,0],[0,0,0,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G3:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A3[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1 ], [ 1 ], [  ], [  ] ], group := Group(()), isGraph := true, names := [ 1 .. 5
],
  order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G3);
false
gap> IsCompleteGraph(G3);
false
gap> IsBipartite(G3);
true
```

In[1754]:= **PathGraphQ[G3]**

Out[1754]= False

In[1751]:= **PlanarGraphQ[G3]**

Out[1751]= True

In[1752]:= **SimpleGraphQ[G3]**

Out[1752]= True

In[1753]:= **TreeGraphQ[G3]**

Out[1753]= False

```
gap> A4:=[[0,1,0,0,0],[1,0,0,0,0],[0,0,0,1,0],[0,0,1,0,0],[0,0,0,0,0]];
[ [ 0, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 1, 0 ], [ 0, 0, 1, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G4:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A4[x][y]=1; end, true);
rec( adjacencies := [ [ 2 ], [ 1 ], [ 4 ], [ 3 ], [  ] ], group := Group(()), isGraph := true, names := [ 1 .. 5 ],
  order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G4);
false
gap> IsCompleteGraph(G4);
false
gap> IsBipartite(G4);
true
```

In[1756]:= **PathGraphQ[G4]**

Out[1756]= False

In[1755]:= **PlanarGraphQ[G4]**

Out[1755]= True

In[1757]:= **SimpleGraphQ[G4]**

Out[1757]= True

In[1758]:= **TreeGraphQ[G4]**

Out[1758]= False

```
gap> A5:=[[0,1,1,1,0],[1,0,0,0,0],[1,0,0,0,0],[1,0,0,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G5:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A5[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1 ], [ 1 ], [ 1 ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G5);
false
gap> IsCompleteGraph(G5);
false
gap> IsBipartite(G5);
true
```

In[1759]:= **PathGraphQ[G5]**

Out[1759]= False

In[1760]:= **PlanarGraphQ[G5]**

Out[1760]= True

In[1761]:= **SimpleGraphQ[G5]**

Out[1761]= True

In[1762]:= **TreeGraphQ[G5]**

Out[1762]= False

```
gap> A6:=[[0,1,1,0,0],[1,0,1,0,0],[1,1,0,0,0],[0,0,0,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G6:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A6[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ], [  ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G6);
false
gap> IsCompleteGraph(G6);
false
gap> IsBipartite(G6);
false
```

In[1764]:= **PathGraphQ[G6]**

Out[1764]= False

In[1765]:= **PlanarGraphQ[G6]**

Out[1765]= True

In[1766]:= **SimpleGraphQ[G6]**

Out[1766]= True

In[1767]:= **TreeGraphQ[G6]**

Out[1767]= False

```
gap> A7:=[[0,1,1,0,0],[1,0,0,1,0],[1,0,0,0,0],[0,1,0,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 0, 1, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 1, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G7:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A7[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1, 4 ], [ 1 ], [ 2 ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G7);
false
gap> IsCompleteGraph(G7);
false
gap> IsBipartite(G7);
true
```

In[1768]:= **PathGraphQ[G7]**

Out[1768]= False

In[1769]:= **PlanarGraphQ[G7]**

Out[1769]= True

In[1770]:= **SimpleGraphQ[G7]**

Out[1770]= True

In[1771]:= **TreeGraphQ[G7]**

Out[1771]= False

```
gap> A8:=[[0,1,1,0,0],[1,0,0,0,0],[1,0,0,0,0],[0,0,0,0,1],[0,0,0,1,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 1 ], [ 0, 0, 0, 1, 0 ] ]
gap> G8:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A8[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1 ], [ 1 ], [ 5 ], [ 4 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G8);
false
gap> IsCompleteGraph(G8);
false
gap> IsBipartite(G8);
true
```

In[1772]:= **PathGraphQ[G8]**

Out[1772]= False

In[1773]:= **PlanarGraphQ[G8]**

Out[1773]= True

In[1774]:= **SimpleGraphQ[G8]**

Out[1774]= True

In[1775]:= **TreeGraphQ[G8]**

Out[1775]= False

```
gap> A9:=[[0,1,1,1,1],[1,0,0,0,0],[1,0,0,0,0],[1,0,0,0,0],[1,0,0,0,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ] ]
gap> G9:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A9[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1 ], [ 1 ], [ 1 ], [ 1 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G9);
false
gap> IsCompleteGraph(G9);
false
gap> IsBipartite(G9);
true
```

In[1776]:= **PathGraphQ[G9]**

Out[1776]= False

In[1777]:= **PlanarGraphQ[G9]**

Out[1777]= True

In[1778]:= **SimpleGraphQ[G9]**

Out[1778]= True

In[1779]:= **TreeGraphQ[G9]**

Out[1779]= True

```
gap> A10:=[[0,1,1,1,0],[1,0,1,0,0],[1,1,0,0,0],[1,0,0,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G10:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A10[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2 ], [ 1 ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G10);
false
gap> IsCompleteGraph(G10);
false
gap> IsBipartite(G10);
false
```

In[1780]:= **PathGraphQ[G10]**

Out[1780]= False

In[1781]:= **PlanarGraphQ[G10]**

Out[1781]= True

In[1782]:= **SimpleGraphQ[G10]**

Out[1782]= True

In[1783]:= **TreeGraphQ[G10]**

Out[1783]= False

```
gap> A11:=[[0,1,1,1,0],[1,0,0,0,1],[1,0,0,0,0],[1,0,0,0,0],[0,1,0,0,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 0, 0, 1 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 0, 1, 0, 0, 0 ] ]
gap> G11:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A11[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 5 ], [ 1 ], [ 1 ], [ 2 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G11);
false
gap> IsCompleteGraph(G11);
false
gap> IsBipartite(G11);
true
```

In[1784]:= **PathGraphQ[G11]**

Out[1784]= False

In[1785]:= **PlanarGraphQ[G11]**

Out[1785]= True

In[1786]:= **SimpleGraphQ[G11]**

Out[1786]= True

In[1787]:= **TreeGraphQ[G11]**

Out[1787]= True

```
gap> A12:=[[0,1,1,0,0],[1,0,1,0,0],[1,1,0,0,0],[0,0,0,0,1],[0,0,0,1,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 0, 0, 0, 0, 1 ], [ 0, 0, 0, 1, 0 ] ]
gap> G12:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A12[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1, 3 ], [ 1, 2 ], [ 5 ], [ 4 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G12);
false
gap> IsCompleteGraph(G12);
false
gap> IsBipartite(G12);
false
```

In[1788]:= **PathGraphQ[G12]**

Out[1788]= False

In[1789]:= **PlanarGraphQ[G12]**

Out[1789]= True

In[1790]:= **SimpleGraphQ[G12]**

Out[1790]= True

In[1791]:= **TreeGraphQ[G12]**

Out[1791]= False

```
gap> A13:=[[0,1,1,0,0],[1,0,0,1,1],[1,0,0,0,0],[0,1,0,0,0],[0,1,0,0,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 0, 1, 1 ], [ 1, 0, 0, 0, 0 ], [ 0, 1, 0, 0, 0 ], [ 0, 1, 0, 0, 0 ] ]
gap> G13:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A13[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1, 4, 5 ], [ 1 ], [ 2 ], [ 2 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G13);
false
gap> IsCompleteGraph(G13);
false
gap> IsBipartite(G13);
true
```

In[1792]:= **PathGraphQ[G13]**

Out[1792]= False

In[1793]:= **PlanarGraphQ[G13]**

Out[1793]= True

In[1794]:= **SimpleGraphQ[G13]**

Out[1794]= True

In[1795]:= **TreeGraphQ[G13]**

Out[1795]= True

```
gap> A14:=[[0,1,1,0,0],[1,0,0,1,0],[1,0,0,1,0],[0,1,1,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 0, 1, 0 ], [ 1, 0, 0, 1, 0 ], [ 0, 1, 1, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G14:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A14[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1, 4 ], [ 1, 4 ], [ 2, 3 ], [  ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G14);
false
gap> IsCompleteGraph(G14);
false
gap> IsBipartite(G14);
true
```

In[1796]:= **PathGraphQ[G14]**

Out[1796]= False

In[1797]:= **PlanarGraphQ[G14]**

Out[1797]= True

In[1798]:= **SimpleGraphQ[G14]**

Out[1798]= True

In[1799]:= **TreeGraphQ[G14]**

Out[1799]= False

```
gap> A15:=[[0,1,1,1,1],[1,0,1,0,0],[1,1,0,0,0],[1,0,0,0,0],[1,0,0,0,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ] ]
gap> G15:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A15[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2 ], [ 1 ], [ 1 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G15);
false
gap> IsCompleteGraph(G15);
false
gap> IsBipartite(G15);
false
```

In[1800]:= **PathGraphQ[G15]**

Out[1800]= False

In[1801]:= **PlanarGraphQ[G15]**

Out[1801]= True

In[1802]:= **SimpleGraphQ[G15]**

Out[1802]= True

In[1803]:= **TreeGraphQ[G15]**

Out[1803]= False

gap> A16:=[[0,1,1,1,0],[1,0,1,1,0],[1,1,0,0,0],[1,1,0,0,0],[0,0,0,0,0]];
[[0, 1, 1, 1, 0], [1, 0, 1, 1, 0], [1, 1, 0, 0, 0], [1, 1, 0, 0, 0], [0, 0, 0, 0, 0]]
gap> G16:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A16[x][y]=1; end, true);
rec(adjacencies := [[2, 3, 4], [1, 3, 4], [1, 2], [1, 2], [ ]], group := Group(()), isGraph := true,
  names := [1 .. 5], order := 5, representatives := [1, 2, 3, 4, 5], schreierVector := [-1, -2, -3, -4, -5]
)
gap> IsRegularGraph(G16);
false
gap> IsCompleteGraph(G16);
false
gap> IsBipartite(G16);
false

In[1804]:= **PathGraphQ[G16]**

Out[1804]= False

In[1805]:= **PlanarGraphQ[G16]**

Out[1805]= True

In[1806]:= **SimpleGraphQ[G16]**

Out[1806]= True

In[1807]:= **TreeGraphQ[G16]**

Out[1807]= False

gap> A17:=[[0,1,1,1,0],[1,0,1,0,1],[1,1,0,0,0],[1,0,0,0,0],[0,1,0,0,0]];
[[0, 1, 1, 1, 0], [1, 0, 1, 0, 1], [1, 1, 0, 0, 0], [1, 0, 0, 0, 0], [0, 1, 0, 0, 0]]
gap> G17:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A17[x][y]=1; end, true);
rec(adjacencies := [[2, 3, 4], [1, 3, 5], [1, 2], [1], [2]], group := Group(()), isGraph := true,
  names := [1 .. 5], order := 5, representatives := [1, 2, 3, 4, 5], schreierVector := [-1, -2, -3, -4, -5]
)
gap> IsRegularGraph(G17);
false
gap> IsCompleteGraph(G17);
false
gap> IsBipartite(G17);
false

In[1808]:= **PathGraphQ[G17]**

Out[1808]= False

In[1809]:= **PlanarGraphQ[G17]**

Out[1809]= True

In[1810]:= **SimpleGraphQ[G17]**

Out[1810]= True

In[1811]:= **TreeGraphQ[G17]**

Out[1811]= False

```
gap> A18:=[[0,1,1,1,0],[1,0,1,0,0],[1,1,0,0,0],[1,0,0,0,1],[0,0,0,1,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 1 ], [ 0, 0, 0, 1, 0 ] ]
gap> G18:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A18[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 3 ], [ 1, 2 ], [ 1, 5 ], [ 4 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G18);
false
gap> IsCompleteGraph(G18);
false
gap> IsBipartite(G18);
false
```

In[1812]:= **PathGraphQ[G18]**

Out[1812]= False

In[1813]:= **PlanarGraphQ[G18]**

Out[1813]= True

In[1814]:= **SimpleGraphQ[G18]**

Out[1814]= True

In[1815]:= **TreeGraphQ[G18]**

Out[1815]= False

```
gap> A19:=[[0,1,1,1,0],[1,0,0,0,1],[1,0,0,0,1],[1,0,0,0,0],[0,1,1,0,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 0, 0, 1 ], [ 1, 0, 0, 0, 1 ], [ 1, 0, 0, 0, 0 ], [ 0, 1, 1, 0, 0 ] ]
gap> G19:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A19[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 5 ], [ 1, 5 ], [ 1 ], [ 2, 3 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G19);
false
gap> IsCompleteGraph(G19);
false
gap> IsBipartite(G19);
true
```

In[1816]:= **PathGraphQ[G19]**

Out[1816]= False

In[1817]:= **PlanarGraphQ[G19]**

Out[1817]= True

In[1818]:= **SimpleGraphQ[G19]**

Out[1818]= True

In[1819]:= **TreeGraphQ[G19]**

Out[1819]= False

```
gap> A20:=[[0,1,1,0,0],[1,0,0,1,0],[1,0,0,0,1],[0,1,0,0,1],[0,0,1,1,0]];
[ [ 0, 1, 1, 0, 0 ], [ 1, 0, 0, 1, 0 ], [ 1, 0, 0, 0, 1 ], [ 0, 1, 0, 0, 1 ], [ 0, 0, 1, 1, 0 ] ]
gap> G20:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A20[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3 ], [ 1, 4 ], [ 1, 5 ], [ 2, 5 ], [ 3, 4 ] ], group := Group(()), isGraph := true,
  names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ], schreierVector := [ -1, -2, -3, -4, -5 ]
)
gap> IsRegularGraph(G20);
true
gap> IsCompleteGraph(G20);
false
gap> IsBipartite(G20);
false
```

In[1820]:= **PathGraphQ[G20]**

Out[1820]= True

In[1821]:= **PlanarGraphQ[G20]**

Out[1821]= True

In[1822]:= **SimpleGraphQ[G20]**

Out[1822]= True

In[1823]:= **TreeGraphQ[G20]**

Out[1823]= False

```
gap> A21:=[[0,1,1,1,1],[1,0,1,1,0],[1,1,0,0,0],[1,1,0,0,0],[1,0,0,0,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 1, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 0 ] ]
gap> G21:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A21[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3, 4 ], [ 1, 2 ], [ 1, 2 ], [ 1 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G21);
false
gap> IsCompleteGraph(G21);
false
gap> IsBipartite(G21);
false
```

In[1824]:= **PathGraphQ[G21]**

Out[1824]= False

In[1825]:= **PlanarGraphQ[G21]**

Out[1825]= True

In[1826]:= **SimpleGraphQ[G21]**

Out[1826]= True

In[1827]:= **TreeGraphQ[G21]**

Out[1827]= False

```
gap> A22:=[[0,1,1,1,1],[1,0,1,0,0],[1,1,0,0,0],[1,0,0,0,1],[1,0,0,1,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 1 ], [ 1, 0, 0, 1, 0 ] ]
gap> G22:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A22[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3 ], [ 1, 2 ], [ 1, 5 ], [ 1, 4 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G22);
false
gap> IsCompleteGraph(G22);
false
gap> IsBipartite(G22);
false
```

In[1828]:= **PathGraphQ[G22]**

Out[1828]= False

In[1829]:= **PlanarGraphQ[G22]**

Out[1829]= True

In[1830]:= **SimpleGraphQ[G22]**

Out[1830]= True

In[1831]:= **TreeGraphQ[G22]**

Out[1831]= False

```
gap> A23:=[[0,1,1,1,0],[1,0,1,1,0],[1,1,0,1,0],[1,1,1,0,0],[0,0,0,0,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 1, 1, 0 ], [ 1, 1, 0, 1, 0 ], [ 1, 1, 1, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
gap> G23:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A23[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ], [  ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G23);
false
gap> IsCompleteGraph(G23);
false
gap> IsBipartite(G23);
false
```

In[1832]:= **PathGraphQ[G23]**

Out[1832]= False

In[1833]:= **PlanarGraphQ[G23]**

Out[1833]= True

In[1834]:= **SimpleGraphQ[G23]**

Out[1834]= True

In[1835]:= **TreeGraphQ[G23]**

Out[1835]= False

```
gap> A24:=[[0,1,1,1,0],[1,0,1,1,0],[1,1,0,0,1],[1,1,0,0,0],[0,0,1,0,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 1, 1, 0 ], [ 1, 1, 0, 0, 1 ], [ 1, 1, 0, 0, 0 ], [ 0, 0, 1, 0, 0 ] ]
gap> G24:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A24[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 5 ], [ 1, 2 ], [ 3 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G24);
false
gap> IsCompleteGraph(G24);
false
gap> IsBipartite(G24);
false
```

In[1836]:= **PathGraphQ[G24]**

Out[1836]= False

In[1837]:= **PlanarGraphQ[G24]**

Out[1837]= True

In[1838]:= **SimpleGraphQ[G24]**

Out[1838]= True

In[1839]:= **TreeGraphQ[G24]**

Out[1839]= False

```
gap> A25:=[[0,1,1,1,0],[1,0,1,0,1],[1,1,0,0,0],[1,0,0,0,1],[0,1,0,1,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 1, 0, 1 ], [ 1, 1, 0, 0, 0 ], [ 1, 0, 0, 0, 1 ], [ 0, 1, 0, 1, 0 ] ]
gap> G25:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A25[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 3, 5 ], [ 1, 2 ], [ 1, 5 ], [ 2, 4 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G25);
false
gap> IsCompleteGraph(G25);
false
gap> IsBipartite(G25);
false
```

In[1840]:= **PathGraphQ[G25]**

Out[1840]= False

In[1841]:= **PlanarGraphQ[G25]**

Out[1841]= True

In[1842]:= **SimpleGraphQ[G25]**

Out[1842]= True

In[1843]:= **TreeGraphQ[G25]**

Out[1843]= False

```
gap> A26:=[[0,1,1,1,0],[1,0,0,0,1],[1,0,0,0,1],[1,0,0,0,1],[0,1,1,1,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 0, 0, 1 ], [ 1, 0, 0, 0, 1 ], [ 1, 0, 0, 0, 1 ], [ 0, 1, 1, 1, 0 ] ]
gap> G26:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A26[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 5 ], [ 1, 5 ], [ 1, 5 ], [ 2, 3, 4 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G26);
false
gap> IsCompleteGraph(G26);
false
gap> IsBipartite(G26);
true
```

In[1844]:= **PathGraphQ[G26]**

Out[1844]= False

In[1845]:= **PlanarGraphQ[G26]**

Out[1845]= True

In[1846]:= **SimpleGraphQ[G26]**

Out[1846]= True

In[1847]:= **TreeGraphQ[G26]**

Out[1847]= False

```
gap> A27:=[[0,1,1,1,1],[1,0,1,1,1],[1,1,0,0,0],[1,1,0,0,0],[1,1,0,0,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 1, 1 ], [ 1, 1, 0, 0, 0 ], [ 1, 1, 0, 0, 0 ], [ 1, 1, 0, 0, 0 ] ]
gap> G27:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A27[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3, 4, 5 ], [ 1, 2 ], [ 1, 2 ], [ 1, 2 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G27);
false
gap> IsCompleteGraph(G27);
false
gap> IsBipartite(G27);
false
```

In[1848]:= **PathGraphQ[G27]**

Out[1848]= False

In[1849]:= **PlanarGraphQ[G27]**

Out[1849]= True

In[1850]:= **SimpleGraphQ[G27]**

Out[1850]= True

In[1851]:= **TreeGraphQ[G27]**

Out[1851]= False

gap> A28:=[[0,1,1,1,1],[1,0,1,1,0],[1,1,0,1,0],[1,1,1,0,0],[1,0,0,0,0]];
[[0, 1, 1, 1, 1], [1, 0, 1, 1, 0], [1, 1, 0, 1, 0], [1, 1, 1, 0, 0], [1, 0, 0, 0, 0]]
gap> G28:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A28[x][y]=1; end, true);
rec(adjacencies := [[2, 3, 4, 5], [1, 3, 4], [1, 2, 4], [1, 2, 3], [1]], group := Group(()),
 isGraph := true, names := [1 .. 5], order := 5, representatives := [1, 2, 3, 4, 5],
 schreierVector := [-1, -2, -3, -4, -5])
gap> IsRegularGraph(G28);
false
gap> IsCompleteGraph(G28);
false
gap> IsBipartite(G28);
false

In[1852]:= **PathGraphQ[G28]**

Out[1852]= False

In[1853]:= **PlanarGraphQ[G28]**

Out[1853]= True

In[1854]:= **SimpleGraphQ[G28]**

Out[1854]= True

In[1855]:= **TreeGraphQ[G28]**

Out[1855]= False

gap> A29:=[[0,1,1,1,1],[1,0,1,1,0],[1,1,0,0,1],[1,1,0,0,0],[1,0,1,0,0]];
[[0, 1, 1, 1, 1], [1, 0, 1, 1, 0], [1, 1, 0, 0, 1], [1, 1, 0, 0, 0], [1, 0, 1, 0, 0]]
gap> G29:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A29[x][y]=1; end, true);
rec(adjacencies := [[2, 3, 4, 5], [1, 3, 4], [1, 2, 5], [1, 2], [1, 3]], group := Group(()),
 isGraph := true, names := [1 .. 5], order := 5, representatives := [1, 2, 3, 4, 5],
 schreierVector := [-1, -2, -3, -4, -5])
gap> IsRegularGraph(G29);
false
gap> IsCompleteGraph(G29);
false
gap> IsBipartite(G29);
false

In[1856]:= **PathGraphQ[G29]**

Out[1856]= False

In[1857]:= **PlanarGraphQ[G29]**

Out[1857]= True

In[1858]:= **SimpleGraphQ[G29]**

Out[1858]= True

In[1859]:= **TreeGraphQ[G29]**

Out[1859]= False

```
gap> A30:=[[0,1,1,1,0],[1,0,1,1,0],[1,1,0,0,1],[1,1,0,0,1],[0,0,1,1,0]];
[ [ 0, 1, 1, 1, 0 ], [ 1, 0, 1, 1, 0 ], [ 1, 1, 0, 0, 1 ], [ 1, 1, 0, 0, 1 ], [ 0, 0, 1, 1, 0 ] ]
gap> G30:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A30[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 5 ], [ 1, 2, 5 ], [ 3, 4 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G30);
false
gap> IsCompleteGraph(G30);
false
gap> IsBipartite(G30);
false
```

In[1860]:= **PathGraphQ[G30]**

Out[1860]= False

In[1861]:= **PlanarGraphQ[G30]**

Out[1861]= True

In[1862]:= **SimpleGraphQ[G30]**

Out[1862]= True

In[1863]:= **TreeGraphQ[G30]**

Out[1863]= False

```
gap> A31:=[[0,1,1,1,1],[1,0,1,1,1],[1,1,0,1,0],[1,1,1,0,0],[1,1,0,0,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 1, 1 ], [ 1, 1, 0, 1, 0 ], [ 1, 1, 1, 0, 0 ], [ 1, 1, 0, 0, 0 ] ]
gap> G31:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A31[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3, 4, 5 ], [ 1, 2, 4 ], [ 1, 2, 3 ], [ 1, 2 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G31);
false
gap> IsCompleteGraph(G31);
false
gap> IsBipartite(G31);
false
```

In[1864]:= **PathGraphQ[G31]**

Out[1864]= False

In[1865]:= **PlanarGraphQ[G31]**

Out[1865]= True

In[1866]:= **SimpleGraphQ[G31]**

Out[1866]= True

In[1867]:= **TreeGraphQ[G31]**

Out[1867]= False

```
gap> A32:=[[0,1,1,1,1],[1,0,1,1,0],[1,1,0,0,1],[1,1,0,0,1],[1,0,1,1,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 1, 0 ], [ 1, 1, 0, 0, 1 ], [ 1, 1, 0, 0, 1 ], [ 1, 0, 1, 1, 0 ] ]
gap> G32:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A32[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3, 4 ], [ 1, 2, 5 ], [ 1, 2, 5 ], [ 1, 3, 4 ] ], group := Group(()),
  isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G32);
false
gap> IsCompleteGraph(G32);
false
gap> IsBipartite(G32);
false
```

In[1868]:= **PathGraphQ[G32]**

Out[1868]= False

In[1869]:= **PlanarGraphQ[G32]**

Out[1869]= True

In[1870]:= **SimpleGraphQ[G32]**

Out[1870]= True

In[1871]:= **TreeGraphQ[G32]**

Out[1871]= False

```
gap> A33:=[[0,1,1,1,1],[1,0,1,1,1],[1,1,0,1,1],[1,1,1,0,0],[1,1,1,0,0]];
[ [ 0, 1, 1, 1, 1 ], [ 1, 0, 1, 1, 1 ], [ 1, 1, 0, 1, 1 ], [ 1, 1, 1, 0, 0 ], [ 1, 1, 1, 0, 0 ] ]
gap> G33:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A33[x][y]=1; end, true);
rec( adjacencies := [ [ 2, 3, 4, 5 ], [ 1, 3, 4, 5 ], [ 1, 2, 4, 5 ], [ 1, 2, 3 ], [ 1, 2, 3 ] ],
  group := Group(()), isGraph := true, names := [ 1 .. 5 ], order := 5, representatives := [ 1, 2, 3, 4, 5 ],
  schreierVector := [ -1, -2, -3, -4, -5 ] )
gap> IsRegularGraph(G33);
false
gap> IsCompleteGraph(G33);
false
gap> IsBipartite(G33);
false
```

In[1872]:= **PathGraphQ[G33]**

Out[1872]= False

In[1873]:= **PlanarGraphQ[G33]**

Out[1873]= True

In[1874]:= **SimpleGraphQ[G33]**

Out[1874]= True

In[1875]:= **TreeGraphQ[G33]**

Out[1875]= False

gap> A34:=[[0,1,1,1,1],[1,0,1,1,1],[1,1,0,1,1],[1,1,1,0,1],[1,1,1,1,0]];
[[0, 1, 1, 1, 1], [1, 0, 1, 1, 1], [1, 1, 0, 1, 1], [1, 1, 1, 0, 1], [1, 1, 1, 1, 0]]
gap> G34:=Graph(Group(()),[1..5],OnPoints, function(x,y) return A34[x][y]=1; end, true);
rec(adjacencies := [[2, 3, 4, 5], [1, 3, 4, 5], [1, 2, 4, 5], [1, 2, 3, 5], [1, 2, 3, 4]],
group := Group(()), isGraph := true, names := [1 .. 5], order := 5, representatives := [1, 2, 3, 4, 5],
schreierVector := [-1, -2, -3, -4, -5])
gap> IsRegularGraph(G34);
true
gap> IsCompleteGraph(G34);
true
gap> IsBipartite(G34);
false

In[1876]:= **PathGraphQ[G34]**

Out[1876]= False

In[1877]:= **PlanarGraphQ[G34]**

Out[1877]= False

In[1878]:= **SimpleGraphQ[G34]**

Out[1878]= True

In[1879]:= **TreeGraphQ[G34]**

Out[1879]= False

#Per ognuno dei grafi su 5 vertici calcola:
il numero cromatico (per definizione il minimo numero di colori necessari a colorare i vertici di un grafo in modo che, presi comunque due vertici adiacenti, essi abbiano diverso colore);
la Girth, ossia la lunghezza del ciclo più breve del grafo;
il raggio, cioè la più breve distanza tra ogni coppia di vertici del grafo;
il diametro, ovvero la più grande distanza tra ogni coppia di vertici del grafo;
il gruppo degli automorfismi.

gap> ChromaticNumber(G1);
1
gap> Girth(G1);
-1

In[1880]:= **GraphRadius[G1]**

Out[1880]= ∞

gap > Diameter (G1);
-1
gap> AutGroupGraph(G1);
Group([(4,5), (3,4), (2,3), (1,2)])

In[1881]:= **GraphAutomorphismGroup[G1]**

Out[1881]= PermutationGroup[
{Cycles[{{4, 5}}], Cycles[{{3, 4}}], Cycles[{{2, 3}}], Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G2);
2
gap> Girth(G2);
-1

In[359]:= **GraphRadius[G2]**

Out[359]= ∞

gap> Diameter(G2);
-1
gap> AutGroupGraph(G2);
Group([(1,2), (4,5), (3,4)])

In[360]:= **GraphAutomorphismGroup[G2]**

Out[360]= PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{3, 4}}], Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G3);
2
gap> Girth(G3);
-1

In[361]:= **GraphRadius[G3]**

Out[361]= ∞

gap> Diameter(G3);
-1
gap> AutGroupGraph(G3);
Group([(2,3), (4,5)])

In[362]:= **GraphAutomorphismGroup[G3]**

Out[362]= PermutationGroup[{Cycles[{{2, 3}}], Cycles[{{4, 5}}]}]

gap> ChromaticNumber(G4);
2
gap> Girth(G4);
-1

In[363]:= **GraphRadius[G4]**

Out[363]= ∞

gap> Diameter(G4);
-1
gap> AutGroupGraph(G4);
Group([(3,4), (1,2), (1,3)(2,4)])

In[364]:= **GraphAutomorphismGroup[G4]**

Out[364]= PermutationGroup[{Cycles[{{3, 4}}], Cycles[{{1, 2}}], Cycles[{{1, 3}, {2, 4}}]}]

gap> ChromaticNumber(G5);
2
gap> Girth(G5);
-1

In[365]:= **GraphRadius[G5]**

Out[365]= ∞

gap> Diameter(G5);
-1
gap> AutGroupGraph(G5);
Group([(3,4), (2,3)])

In[366]:= **GraphAutomorphismGroup[G5]**

Out[366]= PermutationGroup[{Cycles[{{3, 4}}], Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G6);
3
gap> Girth(G6);
3

In[367]:= **GraphRadius[G6]**

Out[367]= ∞

gap> Diameter(G6);
-1
gap> AutGroupGraph(G6);
Group([(2,3), (1,2), (4,5)])

In[368]:= **GraphAutomorphismGroup[G6]**

Out[368]= PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{2, 3}}], Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G7);
2
gap> Girth(G7);
-1

In[369]:= **GraphRadius[G7]**

Out[369]= ∞

gap> Diameter(G7);
-1
gap> AutGroupGraph(G7);
Group([(1,2)(3,4)])

In[370]:= **GraphAutomorphismGroup[G7]**

Out[370]= PermutationGroup[{Cycles[{{1, 2}, {3, 4}}]}]

gap> ChromaticNumber(G8);
2
gap> Girth(G8);
-1

In[371]:= **GraphRadius[G8]**

Out[371]= ∞

gap> Diameter(G8);
-1
gap> AutGroupGraph(G8);
Group([(4,5), (2,3)])

In[372]:= **GraphAutomorphismGroup[G8]**

Out[372]= PermutationGroup[{Cycles[{{2, 3}}], Cycles[{{4, 5}}]}]

gap> ChromaticNumber(G9);
2
gap> Girth(G9);
-1

In[373]:= **GraphRadius[G9]**

Out[373]= 1

gap> Diameter(G9);
2
gap> AutGroupGraph(G9);
Group([(4,5), (3,4), (2,3)])

In[374]:= **GraphAutomorphismGroup[G9]**

Out[374]= PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{3, 4}}], Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G10);
3
gap> Girth(G10);
3

In[375]:= **GraphRadius[G10]**

Out[375]= ∞

gap> Diameter(G10);
-1
gap> AutGroupGraph(G10);
Group([(2,3)])

In[376]:= **GraphAutomorphismGroup[G10]**

Out[376]= PermutationGroup[{Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G11);
2
gap> Girth(G11);
-1

In[377]:= **GraphRadius[G11]**

Out[377]= 2

gap> Diameter(G11);
3
gap> AutGroupGraph(G11);
Group([(3,4)])

In[378]:= **GraphAutomorphismGroup[G11]**

Out[378]= PermutationGroup[{Cycles[{{3, 4}}]}]

gap> ChromaticNumber(G12);
3
gap> Girth(G12);
3

In[379]:= **GraphRadius[G12]**

Out[379]= ∞

gap> Diameter(G12);
-1
gap> AutGroupGraph(G12);
Group([(2,3), (1,2), (4,5)])

In[380]:= **GraphAutomorphismGroup[G12]**

Out[380]= PermutationGroup[{Cycles[{{2, 3}}], Cycles[{{1, 2}}], Cycles[{{4, 5}}]}]

gap> ChromaticNumber(G13);
2
gap> Girth(G13);
-1

In[381]:= **GraphRadius[G13]**

Out[381]= 2

gap> Diameter(G13);
3
gap> AutGroupGraph(G13);
Group([(4,5)])

In[382]:= **GraphAutomorphismGroup[G13]**

Out[382]= PermutationGroup[{Cycles[{{4, 5}}]}]

gap> ChromaticNumber(G14);
2
gap> Girth(G14);
4

In[48]:= **GraphRadius[G14]**

Out[48]= ∞

gap> Diameter(G14);
-1
gap> AutGroupGraph(G14);
Group([(2,3), (1,2)(3,4)])

In[49]:= **GraphAutomorphismGroup[G14]**

Out[49]= PermutationGroup[{Cycles[{{2, 3}}], Cycles[{{1, 2}, {3, 4}}]}]

gap> ChromaticNumber(G15);
3
gap> Girth(G15);
3

In[50]:= **GraphRadius[G15]**

Out[50]= 1

gap> Diameter(G15);
2
gap> AutGroupGraph(G15);
Group([(2,3), (4,5)])

In[51]:= **GraphAutomorphismGroup[G15]**

Out[51]= PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G16);
3
gap> Girth(G16);
3

In[52]:= **GraphRadius[G16]**

Out[52]= ∞

gap> Diameter(G16);
-1
gap> AutGroupGraph(G16);
Group([(1,2), (3,4)])

In[53]:= **GraphAutomorphismGroup[G16]**

Out[53]= PermutationGroup[{Cycles[{{3, 4}}], Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G17);
3
gap> Girth(G17);
3

In[54]:= **GraphRadius[G17]**

Out[54]= 2

gap> Diameter(G17);
3
gap> AutGroupGraph(G17);
Group([(1,2)(4,5)])

In[55]:= **GraphAutomorphismGroup[G17]**

Out[55]= PermutationGroup[{Cycles[{{1, 2}, {4, 5}}]}]

gap> ChromaticNumber(G18);
3
gap> Girth(G18);
3

In[56]:= **GraphRadius[G18]**

Out[56]= 2

gap> Diameter(G18);
3
gap> AutGroupGraph(G18);
Group([(2,3)])

In[57]:= **GraphAutomorphismGroup[G18]**

Out[57]= PermutationGroup[{Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G19);
2
gap> Girth(G19);
4

In[58]:= **GraphRadius[G19]**

Out[58]= 2

gap> Diameter(G19);
3
gap> AutGroupGraph(G19);
Group([(2,3)])

In[59]:= **GraphAutomorphismGroup[G19]**

Out[59]= PermutationGroup[{Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G20);
3
gap> Girth(G20);
5

In[60]:= **GraphRadius[G20]**

Out[60]= 2

gap> Diameter(G20);
2
gap> AutGroupGraph(G20);
Group([(2,3)(4,5), (1,2)(3,4)])

In[61]:= **GraphAutomorphismGroup[G20]**

Out[61]= PermutationGroup[{Cycles[{{2, 3}, {4, 5}}], Cycles[{{1, 2}, {3, 4}}]}]

gap> ChromaticNumber(G21);
3
gap> Girth(G21);
3

In[62]:= **GraphRadius[G21]**

Out[62]= 1

gap> Diameter(G21);
2
gap> AutGroupGraph(G21);
Group([(3,4)])

In[63]:= **GraphAutomorphismGroup[G21]**

Out[63]= PermutationGroup[{Cycles[{{3, 4}}]}]

gap> ChromaticNumber(G22);
3
gap> Girth(G22);
3

In[64]:= **GraphRadius[G22]**

Out[64]= 1

gap> Diameter(G22);
2
gap> AutGroupGraph(G22);
Group([(4,5), (2,3), (2,4)(3,5)])

In[65]:= **GraphAutomorphismGroup[G22]**

Out[65]= PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{2, 3}}], Cycles[{{2, 4}, {3, 5}}]}]

gap> ChromaticNumber(G23);
4
gap> Girth(G23);
3

In[66]:= **GraphRadius[G23]**

Out[66]= ∞

gap> Diameter(G23);
-1
gap> AutGroupGraph(G23);
Group([(3,4), (2,3), (1,2)])

In[67]:= **GraphAutomorphismGroup[G23]**

Out[67]= PermutationGroup[{Cycles[{{3, 4}}], Cycles[{{2, 3}}], Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G24);
3
gap> Girth(G24);
3

In[68]:= **GraphRadius[G24]**

Out[68]= 2

gap> Diameter(G24);
3
gap> AutGroupGraph(G24);
Group([(1,2)])

In[69]:= **GraphAutomorphismGroup[G24]**

Out[69]= PermutationGroup[{Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G25);
3
gap> Girth(G25);
3

In[71]:= **GraphRadius[G25]**

Out[71]= 2

gap> Diameter(G25);
2
gap> AutGroupGraph(G25);
Group([(1,2)(4,5)])

In[72]:= **GraphAutomorphismGroup[G25]**

Out[72]= PermutationGroup[{Cycles[{{1, 2}, {4, 5}}]}]

gap> ChromaticNumber(G26);
2
gap> Girth(G26);
4

In[73]:= **GraphRadius[G26]**

Out[73]= 2

gap> Diameter(G26);
2
gap> AutGroupGraph(G26);
Group([(1,5), (3,4), (2,3)])

In[74]:= **GraphAutomorphismGroup[G26]**

Out[74]= PermutationGroup[{Cycles[{{1, 5}}], Cycles[{{3, 4}}], Cycles[{{2, 3}}]}]

gap> ChromaticNumber(G27);
3
gap> Girth(G27);
3

In[75]:= **GraphRadius[G27]**

Out[75]= 1

gap> Diameter(G27);
2
gap> AutGroupGraph(G27);
Group([(1,2), (4,5), (3,4)])

In[76]:= **GraphAutomorphismGroup[G27]**

Out[76]= PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{3, 4}}], Cycles[{{1, 2}}]}]

gap> ChromaticNumber(G28);
4
gap> Girth(G28);
3

In[77]:= **GraphRadius[G28]**

Out[77]= 1

gap> Diameter(G28);
2
gap> AutGroupGraph(G28);
Group([(3,4), (2,3)])

In[78]:= **GraphAutomorphismGroup[G29]**

Out[78]= PermutationGroup[{Cycles[{{2, 3}, {4, 5}}]}]

gap> ChromaticNumber(G29);
3
gap> Girth(G29);
3

In[79]:= **GraphRadius[G29]**

Out[79]= 1

gap> Diameter(G29);
2
gap> AutGroupGraph(G29);
Group([(2,3)(4,5)])

In[80]:=
```
GraphAutomorphismGroup[G29]
```
Out[80]=
```
PermutationGroup[{Cycles[{{2, 3}, {4, 5}}]}]
```

```
gap> ChromaticNumber(G30);
3
gap> Girth(G30);
3
```

In[81]:=
```
GraphRadius[G30]
```
Out[81]=
```
2
```

```
gap> Diameter(G30);
2
gap> AutGroupGraph(G30);
Group([ (1,2), (3,4) ])
```

In[82]:=
```
GraphAutomorphismGroup[G30]
```
Out[82]=
```
PermutationGroup[{Cycles[{{3, 4}}], Cycles[{{1, 2}}]}]
```

```
gap> ChromaticNumber(G31);
4
gap> Girth(G31);
3
```

In[83]:=
```
GraphRadius[G31]
```
Out[83]=
```
1
```

```
gap> Diameter(G31);
2
gap> AutGroupGraph(G31);
Group([ (1,2), (3,4) ])
```

In[84]:=
```
GraphAutomorphismGroup[G31]
```
Out[84]=
```
PermutationGroup[{Cycles[{{1, 2}}], Cycles[{{3, 4}}]}]
```

```
gap> ChromaticNumber(G32);
3
gap> Girth(G32);
3
```

In[85]:=
```
GraphRadius[G32]
```
Out[85]=
```
1
```

```
gap> Diameter(G32);
2
gap> AutGroupGraph(G32);
Group([ (3,4), (2,3)(4,5) ])
```

In[86]:=
```
GraphAutomorphismGroup[G32]
```
Out[86]=
```
PermutationGroup[{Cycles[{{3, 4}}], Cycles[{{2, 3}, {4, 5}}]}]
```

```
gap> ChromaticNumber(G33);
4
gap> Girth(G33);
3
```

In[87]:=
```
GraphRadius[G33]
```
Out[87]=
```
1
```

```
gap> Diameter(G33);
2
gap> AutGroupGraph(G33);
Group([ (2,3), (1,2), (4,5) ])
```

In[88]:=
```
GraphAutomorphismGroup[G33]
```
Out[88]=
```
PermutationGroup[{Cycles[{{4, 5}}], Cycles[{{2, 3}}], Cycles[{{1, 2}}]}]
```

```
gap> ChromaticNumber(G34);
5
gap> Girth(G34);
3
```

In[89]:=
```
GraphRadius[G34]
```
Out[89]=
```
1
```

```
gap> Diameter(G34);
1
gap> AutGroupGraph(G34);
Group([ (4,5), (3,4), (2,3), (1,2) ])
```

In[90]:=
```
GraphAutomorphismGroup[G34]
```
Out[90]=
```
PermutationGroup[
 {Cycles[{{4, 5}}], Cycles[{{3, 4}}], Cycles[{{2, 3}}], Cycles[{{1, 2}}]}]
```